

A numerical method for elastic and viscoelastic dynamic fracture problems in homogeneous and bimaterial systems

P. H. Geubelle

20

Abstract We present a summary of recent advances in the development of an efficient numerical scheme to be used in the investigation of a wide range of 2D and 3D dynamic fracture problems. The numerical scheme, which is based on a spectral representation of the boundary integral relations, can be applied to homogeneous and interfacial dynamic fracture problems involving planar cracks and faults of arbitrary shapes buried in elastic and viscoelastic media. Spontaneous propagation of the crack is achieved by combining the elastodynamic integral relations with a stress-based cohesive failure model. The objective of this paper is to present some of the major differences existing between the various formulations within the simpler 2D scalar framework of anti-plane shear (mode III) loading conditions. Examples are presented to illustrate some capabilities of the method.

1

Introduction

Except in a very small number of cases involving simplified (semi-infinite) geometries and (uniform mode III) loading conditions, computational techniques must be used in the analysis of *spontaneous* crack propagation, for which the crack path and velocity are not known a priori and are part of the solution itself. Among the numerical proposed over the past few years, the spectral method first introduced by Perrin et al. (1994) and generalized by Geubelle and Rice (1995) appears to be one of the most performant tools currently available to investigate a wide range of 2D and 3D dynamic fracture problems involving planar cracks and faults of arbitrary shapes and subjected to any combination of loading conditions. The numerical scheme, which is a special form of the boundary integral relation existing between the tractions on the fracture

plane and the corresponding displacement discontinuities, has proven to provide a very precise and efficient way to investigate the response of stationary planar cracks subjected to impact-like loading conditions and the evolution of spontaneously propagating fast cracks. Efficiency is one of the method's main advantages and is associated with the use of simple explicit time stepping schemes and Fast Fourier Transform (FFT) algorithms and with its adaptability to massively parallel computing environments (Geubelle 1994), allowing for the investigation of very large problems.

Another advantage of the method is the fact that it can be relatively easily extended to other 2D and 3D geometries and material systems: while the initial formulation described in the aforementioned papers was aimed at the investigation of various dynamic fracture problems involving planar cracks in homogeneous linearly elastic media, recent work has extended the spectral formulation to dynamic debonding problems (Geubelle and Breitenfeld 1996; Breitenfeld and Geubelle 1996) and to the dynamic failure of viscoelastic media (Geubelle et al. 1996; Danyluk et al. 1996).

The objective of the present paper is to summarize these recent advances and to contrast the new interfacial and viscoelastic spectral schemes with the original homogeneous linearly elastic formulation within the framework of antiplane shear (mode III) loading conditions. This 2D scalar case was chosen because, despite its simplicity, it contains most of the characteristics of the various spectral formulations for the more complex 3D vectorial problems and is therefore more adequate to compare the various forms of the spectral scheme. Details on the derivation of the methods are left out of this paper and can be found in the papers listed above. The present paper is divided into three parts: in Section 2, the homogeneous linearly elastic mode III formulation is presented, together with a brief summary of an important improvement recently proposed by Cochard and Rice (1996). Section 3 is dedicated to the dynamic debonding problem while the final section discusses the viscoelastic formulation of the spectral scheme.

2

Spectral formulation for homogeneous elastic problems

As mentioned earlier, the first version of the spectral scheme was developed and implemented by Perrin et al. (1994) who investigated the problem of a fault buried in an infinite homogeneous linearly elastic medium and subjected to dynamic anti-plane shear loading conditions. The

Communicated by P. E. O'Donoghue, M. D. Gilchrist and K. B. Broberg, 6 January 1997

P. H. Geubelle
Department of Aeronautical and Astronautical Engineering,
University of Illinois at Urbana-Champaign,
Urbana, Illinois, USA

The author wishes to acknowledge the assistance of his co-workers M. S. Breitenfeld, M. J. Danyluk and H. H. Hilton in the development of the bimaterial and viscoelastic versions of the spectral scheme and his collaboration with A. Cochard, J. W. Morrissey and J. R. Rice on various aspects of the initial development of the numerical scheme.

main focus of their analysis was to take advantage of the versatility of the spectral scheme to study the influence of various state- and rate-dependent friction models on the spontaneous propagation and healing of a seismic pulse along the fracture plane. The optimum implementation of the spectral formulation and the associated precision and stability characteristics were later detailed by Morrissey and Geubelle (1995), who also showed how the method can be used to extract the dynamic stress intensity factor from the near-tip values of the displacement discontinuity (or slip).

The spectral scheme is based on a special form of the elastodynamic boundary integral equation and therefore relates the traction stresses acting on the fracture plane $y = 0$ to the associated displacement discontinuities. In the simpler 2D homogeneous elastic mode III case characterized by the single out-of-plane displacement component $w(x, y, t)$, the traction stress $\tau(x, t) = \sigma_{yz}(x, y = 0, t)$ can be readily shown to be related to the slip $\delta(x, t) = w(x, y = 0^+, t) - w(x, y = 0^-, t) = 2w(x, y = 0^+, t)$ on the fracture plane as

$$\tau(x, t) = \tau^o(x, t) - \frac{\mu}{2c_s} \frac{\partial \delta(x, t)}{\partial t} + f(x, t) . \quad (1)$$

In (1), μ and c_s denote the shear modulus and shear wave speed of the surrounding linearly elastic homogeneous infinite medium, respectively; $\tau^o(x, t)$ is the externally applied anti-plane shear stress, i.e., the stress that would exist on the fracture plane $y = 0$ if no crack was present; the second right-hand-side term corresponds to the instantaneous response of the material; the last term incorporates the dynamic effects associated with the non-uniform motion of the fracture surfaces. In conventional boundary integral formulations, this term takes the form of a double convolution integral over the portion of the (x, t) plane centered at the point of observation and expanding in a cone-like shape to include all the surrounding points which have influenced the current value of the dynamic stress. In the spectral formulation, however, the convolution term is expressed explicitly in the spectral domain as a single integral formulation over the past history of the Fourier coefficients of the displacement discontinuity as

$$\{f(x, t), \delta(x, t)\} = \{F(t; q), D(t; q)\} e^{iqx} , \quad (2)$$

where

$$F(t; q) = -\frac{\mu|q|}{2} \int_{-\infty}^t C_{\text{III}}^{\text{EL}}(|q|c_s t') D(t - t'; q) |q|c_s dt' . \quad (3)$$

In the mode III elastic homogeneous problem, the convolution kernel takes the simple expression

$$C_{\text{III}}^{\text{EL}}(T) = J_1(T)/T , \quad (4)$$

where $J_1(T)$ denotes the Bessel function of the first kind. Unlike most boundary integral formulations, the convolution kernel is non-singular, due to the extraction of the instantaneous response in (1). The kernel, which is represented by the solid curve in Fig. 1, is an oscillating and rapidly decaying function.

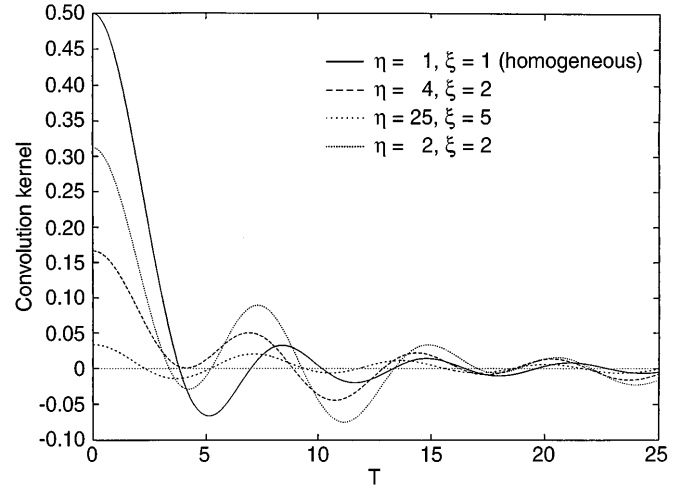


Fig. 1. Convolution kernel $C_{\text{III}}^{\text{EL}}(T; \eta, \xi)$ for the combined bimaterial elastic mode III spectral formulation

Instead of the slip-based expression (3) of the convolution term, another formulation based on a convolution performed on the past slip rate history can be derived by integrating (3) by parts and extracting the long-term static limit. This equivalent velocity-based formulation is of interest in situations characterized by the succession of long periods of slow loading of a stationary crack interrupted by sudden bursts of rapid crack propagation, such as those encountered in the simulation of earthquake cycles (Rice and Ben-Zion 1996) and crack arrest (Morrissey and Geubelle 1995).

The numerical implementation of the spectral method is straightforward and has been described in great details by Morrissey and Geubelle (1995). A portion X of the fracture plane is discretized by a uniform grid and the solution is approximated by a Fourier series representation with period X . In other words, for the n^{th} spectral mode, the mode number q appearing in (2) is replaced by $2\pi n/X$. The relation between spatial and spectral domains is obtained efficiently through a FFT algorithm using the pre-defined grid points as sampling points. Efficiency is also achieved by using an explicit time stepping scheme to derive the displacement distribution from the slip rate distribution obtained in (1).

To simulate the spontaneous propagation of mode III cracks, a stress-based cohesive failure model must be introduced, which relates the strength on the fracture plane to the current and, possibly, past values of the slip and slip rate, and to the position on the fracture plane if the latter is characterized by a heterogeneous toughness distribution (Geubelle and Rice 1995). The distinction between the crack, the cohesive zone and the surrounding “intact” medium is achieved very simply by comparing the value of the dynamic stress (i.e., the sum of the externally applied stress $\tau^o(x, t)$ and the convolution term $f(x, t)$ in (1)) with the current value of the strength: failure takes place (i.e., the slip rate will be non-zero in (1)) only if the strength is lower than the dynamic stress. The crack itself corresponds to the region in which the current strength value is zero. In the case of shear-dominated loading conditions

such as the mode III case discussed in the present paper, friction between the fracture surfaces may play a major role in the overall propagation and arrest behavior of the crack. As discussed by Perrin et al. (1994), a wide range of friction models can be incorporated in the spectral scheme.

The Fourier series representation of the various stress and displacement quantities on the fracture plane introduces a spatial replication of the fracture event. Instead of simulating the dynamic behavior of a single crack or fault, the method provides the solution associated with the dynamic behavior of a period array of cracks, the spatial periodicity being given by the size X of the discretized portion of the fracture plane. This artificial periodicity might be useful in certain problem, such as that of the dynamic interaction of a rapidly moving crack with a row of asperities (i.e., regions of higher fracture toughness) discussed by Geubelle and Rice (1995). But, in most cases, the focus is on the dynamic behavior of a single crack and a large portion of the fracture plane must be discretized in order to limit the interaction associated with the presence of the artificially introduced neighboring cracks. This limitation has been the object of a recent paper by Cochard and Rice (1996) who managed to keep the same spectral formulation of the elastodynamic relations and remove the artificial replication of the fracture event. They obtained this important result by taking the size X of the discretized portion of the fracture plane as twice the crack length L and by using a different convolution kernel. But, unlike in the original spectral formulation in which the convolution kernel $C_{\text{III}}^{\text{EL}}(T)$ is the same for all spectral modes and the dependence on the mode number q appears only in its argument, the convolution kernel appearing in Cochard and Rice's replication-free formulation is different for each spectral mode. The n^{th} mode time-dependent Fourier coefficient $F(t; n)$ of the convolution term is now related to the corresponding Fourier coefficients $D(t; n)$ of the displacement discontinuity through the integral relation

$$F(t; n) = -\frac{\mu\pi^2 n^2}{2L} \int_{-\infty}^t \bar{C}_{\text{III}}^{\text{EL}}\left(\frac{c_s(t-t')}{L}; n\right) D(t'; n) \frac{c_s}{L} dt', \quad (5)$$

where the convolution kernel $\bar{C}_{\text{III}}^{\text{EL}}(T; n)$ of the replication-free spectral formulation is expressed in terms of the original kernel $C_{\text{III}}^{\text{EL}}(T)$ as

$$\bar{C}_{\text{III}}^{\text{EL}}(T; n) = \frac{1}{\pi} \int_{-\infty}^{\infty} C_{\text{III}}^{\text{EL}}(|u|T) \frac{\sin(u - n\pi)}{u - n\pi} du \quad (6)$$

and must be computed numerically.

Examples of simulations involving a mode III crack in a homogeneous linearly elastic material will be given in the next section, as particular case of the bimaterial problem.

3 Spectral formulation for elastic interfacial problems

Two basic approaches are possible to simulate dynamic debonding problems. The first one, referred to as *the combined spectral formulation* is in essence very similar to the homogeneous formulation described in the previous section. It combines the elastodynamic response of both

half spaces and the interface continuity conditions in a single "bimaterial" boundary integral equation involving the crack opening displacement (or displacement discontinuity) and the dynamic stresses acting on the fracture plane. In the simpler scalar mode III problem discussed by Geubelle and Breitenfeld (1996), the combined spectral formulation takes the familiar form

$$\tau(x, t) = \tau^o(x, t) - \frac{\mu^+}{c_s^+} \frac{\xi}{\xi + \eta} \frac{\partial \delta(x, t)}{\partial t} + f^c(x, t) \quad (7)$$

where the superscripts $()^+$ and $()^-$ denote quantities associated with the top and bottom half spaces, respectively; $\xi = c_s^+/c_s^-$ and $\eta = \mu^+/\mu^-$ are non-dimensional mismatch parameters. The convolution term is also almost identical to its homogeneous counterpart, and is expressed in the spectral domain as

$$F^c(t; q) = -\frac{\mu^+ |q|}{2} \int_{-\infty}^t C_{\text{III}}^{\text{EL}^c}(|q|c_s^+ t'; \eta, \xi) D(t-t'; q) |q|c_s^+ dt' \quad (8)$$

In the latter relation, $C_{\text{III}}^{\text{EL}^c}(T; \eta, \xi)$ denotes the convolution kernel of the elastic bimaterial mode III problem. Its expression is somewhat more complex than its homogeneous counterpart and its evaluation requires the use of a numerical Laplace inversion scheme, except in the special homogeneous case ($\eta = \xi = 1$) for which we recover the simple expression (4). The convolution kernel is shown in Fig. 1 for various values of the mismatch parameters η and ξ .

The combined approach has the main advantage of combining the elastodynamic responses of both half spaces in a single integral relation, thereby providing substantial saving in both computational effort and storage. However, its extension to the more complex 2D in-plane (modes I/II) and 3D bimaterial situations characterized by an intricate combination of tensile and shear displacement discontinuities is extremely complicated. This fact has motivated the development of the second approach to bimaterial problems.

In the second approach, referred to as the *independent spectral formulation*, the dynamic responses of both half spaces are modeled separately using two independent homogeneous boundary integrals, and are then joined through appropriate continuity conditions along the interface $y = 0$. In other words, instead of using the displacement jump $\delta(x, t)$ as in the combined formulation, we write two integral relations in terms of the displacement of the surfaces of the top and bottom half spaces $w^\pm(x, t) = w(x, y = 0^\pm, t)$ as in

$$\tau(x, t) = \tau^o(x, t) \mp \frac{\mu^\pm}{2c_s^\pm} \frac{\partial w^\pm(x, t)}{\partial t} + f^\pm(x, t) \quad (9)$$

The spectral form of the convolution term is then almost identical to the homogeneous counterpart

$$F^\pm(t; q) = \mp \mu^\pm |q| \int_{-\infty}^t C_{\text{III}}^{\text{EL}^{\text{in}}}(|q|c_s^\pm t') W^\pm(t-t'; q) |q|c_s^\pm dt' \quad (10)$$

in which $W^\pm(t; q)$ denotes the Fourier coefficient of the out-of-plane displacement $w^\pm(x, t)$ along the fracture plane. The convolution kernel of the independent spectral formulation $C_{III}^{EL, in}$ is thus the same as the homogeneous kernel and is given by (4).

The main disadvantage of this second approach is the additional computational costs associated with the solution of two integral equations. However, by independently computing the dynamic response of the two homogeneous components of the bimaterial system, the independent formulation is much simpler than the combined one, and it allows for the simulation of any bimaterial material combination (such as elastic/viscoelastic, ...). Furthermore, unlike relations (7) and (8) which involve the displacement discontinuity $\delta(x, t)$, (9) and (10) are expressed in terms of the displacement of interface plane. Therefore, the independent formulation provides additional information relative to the motion of the interface itself in the uncracked region under the effect of elastic waves travelling at different speeds in the two adjacent half spaces. This fact will be illustrated below.

The implementation of both bimaterial spectral schemes is similar to that of the homogeneous case. A wide range of stress-based cohesive laws can be introduced to model the failure of the interface in the simulation of spontaneous dynamic debonding. Unlike in the 2D in-plane and fully 3D situations for which the independent formulation appears to be much more stable, the two approaches have, in the mode III case, the same precision and stability characteristics and provide identical results.

Figures 2–4 illustrate two typical applications of the bimaterial spectral scheme. In the first one, a stationary interfacial crack of length $2a$ is suddenly subjected to a uniform anti-plane shear loading τ_o . In order to minimize the effect of the artificially introduced neighboring cracks, the domain X of investigation is four times larger than the crack size. It is discretized by a uniform grid composed of 1024 elements of equal length Δx . The time step used in the explicit stepping scheme is chosen as $\Delta t = \Delta x / 2c_s^+$,

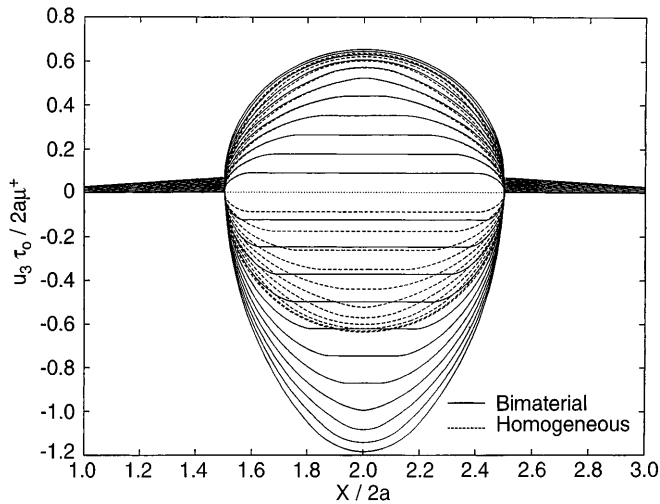


Fig. 2. Evolution of the deformed shape of a crack of length $2a$ subjected to a sudden uniform mode III load τ_o in the homogeneous (dashed) and bimaterial case (solid curve)

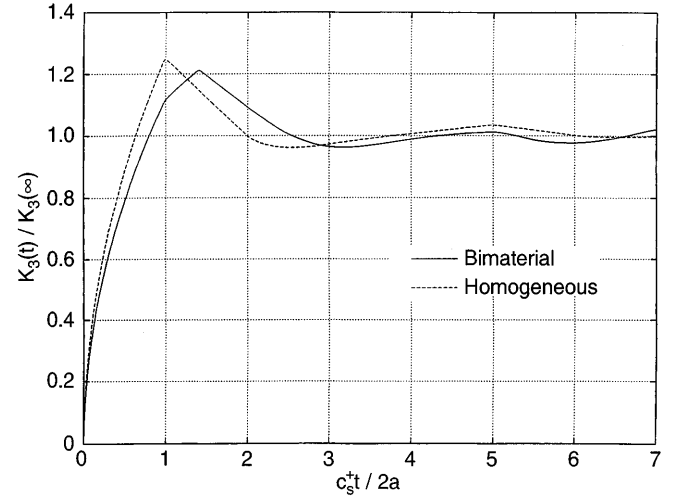


Fig. 3. Evolution of the dynamic stress intensity factor (normalized by its static limit) for the interfacial problem presented in Fig. 2

i.e., it takes two time steps for the fastest shear wave to travel one grid point spacing. Figure 2 presents the evolution of the crack deformed shape in the homogeneous case (dashed line) and the bimaterial situation (with $\eta = \zeta^2 = 2$) (solid curve). The various curves are separated by 100 time steps. In the homogeneous case, the deformation are symmetric as the shear waves travel at the same speed along the fracture surfaces, and the interface in the uncracked region does not move. In the bimaterial case, however, the waves travel at different speeds along the top and bottom crack faces as illustrated by the time it takes for the waves emitted at the crack tips to reach the center of the crack. The higher compliance of the bottom material is also apparent in the deformed shape of the crack. Note finally how the asymmetry affects the motion of the interface. The latter has a “damping” effect on the evolution of the dynamic stress intensity factor $K_3(t)$ ex-

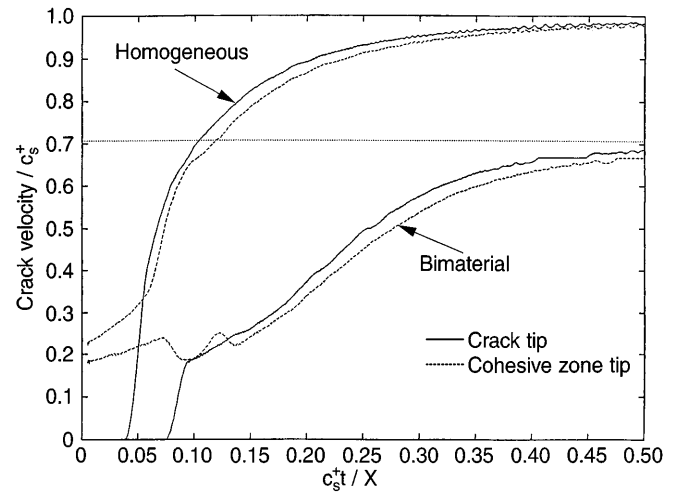


Fig. 4. Spontaneous crack propagation: evolution of the velocity of the leading and trailing edges of the cohesive zone in the homogeneous and bimaterial cases

tracted from the near-tip distribution of the slip and shown in Fig. 3: the dynamic overshoot characteristic of this type of dynamic loading problem is lower in the bimaterial case than in the homogeneous one.

In the second illustrative problem, we investigate the issue of limiting debonding speed under mode III conditions. An asymptotic investigation by Yu and Yang (1994) has shown that, under transonic steady-state conditions, i.e., when the constant crack speed exceeds the slower shear wave speed but is less than the higher shear wave speed of the bimaterial system, no energy flows from the surrounding medium to the crack tip. The flow of energy takes place through the interface from the “elliptic” (stiffer) material to the “hyperbolic” (more compliant) one. This result indicates that, since energy is needed to create a new interfacial fracture surface, the maximum spontaneous debonding speed under anti-plane shear conditions is bounded by the lower shear wave speed of the bimaterial system.

This fact is confirmed by the analysis shown in Fig. 4 which presents the result of a simulation of a spontaneously propagating mode III crack using the spectral scheme. In order to model the failure of the interface, a simple cohesive model was used in which the interfacial strength is assumed to decrease linearly with the slip, from an intact value τ_c corresponding to $\delta = 0$, to zero at a critical value δ_c of the slip, beyond which complete failure is assumed. The decohesion energy was thus given by $\tau_c \delta_c / 2$. The velocities of both the leading edge (referred to as the cohesive zone tip) and trailing edge (i.e., the actual crack tip) of the cohesive zone are presented for a homogeneous and bimaterial (with $\eta = \xi^2 = 2$) geometries. At the beginning of the simulation, the cohesive zone starts to propagate at an increasing velocity while the crack tip itself remains stationary until sufficient slip has developed. At that point, the crack tip jumps to a very high velocity in an attempt to catch up with the cohesive zone tip. After some transients, the speed of the crack tip remains consistently higher than that of the cohesive zone tip, indicating a continuous shrinking of the cohesive zone size as the debonding asymptotically approaches its limiting speed.

4

Spectral formulation for viscoelastic problems

In this last section, we extend the spectral formulation to mode III dynamic fracture problems in viscoelastic media. The basic steps of the derivation are very similar to those used in the elastic situation. The only difference is associated with the time dependence of the mechanical response in shear of the surrounding medium, the shear relaxation modulus of which is written in the usual Prony exponential series

$$\mu(t) = \left[\mu_\infty + \sum_{n=1}^N \mu_n e^{-t/\tau_n} \right] H(t) , \quad (11)$$

in which $H(t)$ is the Heaviside step function and μ_∞ is the fully relaxed value of the shear modulus. The final expression of the viscoelastic spectral formulation is

$$\tau(x, t) = \tau^o(x, t) - \frac{\mu_o}{2c_{so}} \frac{\partial \delta(x, t)}{\partial t} + Q\delta(x, t) + f(x, t) , \quad (12)$$

where $\mu_o = \mu_\infty + \sum \mu_n$ is the instantaneous value of the shear modulus; $c_{so} = \sqrt{\mu_o/\rho}$ is the associated shear wave speed and Q is a material constant given by

$$Q = \frac{\mu_o}{4c_{so}\tau^*} \sum_{n=1}^N \frac{\mu_n \tau^*}{\mu_o \tau_n} . \quad (13)$$

In the latter, τ^* is an arbitrarily chosen characteristic relaxation time of the material. The new term $Q\delta(x, t)$ in (12), which involves the current value of the slip, is characteristic of the viscoelastic problem and disappears in the elastic limit (for which all time constants τ_n tend to infinity). The convolution term $f(x, t)$ is again expressed in the spectral domain as

$$F(t; q) = -\frac{\mu_o |q|}{2} \int_{-\infty}^t C_{\text{III}}^{\text{VE}}(|q|c_{so}t') D(t-t'; q) |q|c_{so} dt' \\ F(t; 0) = -\frac{\mu_o}{2c_{so}\tau^*} \int_{-\infty}^t C_0^{\text{VE}}\left(\frac{t'}{\tau^*}\right) D(t-t'; 0) \frac{dt'}{\tau^*} \quad (14)$$

The convolution kernels $C_{\text{III}}^{\text{VE}}(T)$ and $C_0^{\text{VE}}(T)$ combine the viscoelastic and dynamic response. Note that, unlike in the elastic case where it does not contribute to the convolution integral, the constant (of zeroth) spectral mode $q = 0$ has a non-vanishing contribution and represents the time-dependent uniform motion of a viscoelastic half-space.

The convolution kernels depend on the material parameters used in the Prony series representation of the shear response and must, in most cases, be computed numerically from their Laplace transform expression. For the Standard Linear Solid (SLS) class of viscoelastic materials, the relaxation response is given by

$$\mu(t) = \mu_\infty \left[1 + \xi \exp\left(\frac{-(1+\xi)t}{\tau}\right) \right] H(t) \quad (15)$$

and is characterized by only three material parameters μ_∞ , $\xi = \mu_o/\mu_\infty - 1$ and the unique relaxation time constant $\tau^* = \tau$, the constant mode convolution kernel is given by

$$C_0^{\text{VE}}(T) = \frac{\xi}{2} e^{-(1+\xi/2)T} \left[(1+\xi) \left\{ I_0\left(\frac{\xi T}{2}\right) - I_1\left(\frac{\xi T}{2}\right) \right\} - \frac{I_1(\xi T/2)}{T} \right] \quad (16)$$

where I_0 and I_1 are modified Bessel functions. The convolution kernel for the non-zero spectral modes is shown in Fig. 5 for various values of $b = 1/|q|c_{so}\tau$. Unlike in the initial elastic formulation in which the convolution kernel is mode independent*, the latter depends strongly, in the

* Recall that, in the replication-free formulation recently presented by Cochard and Rice (1996) and summarized by (5)-(6), the new convolution kernel $\overline{C}_{\text{III}}^{\text{EL}}$ is mode-dependent.

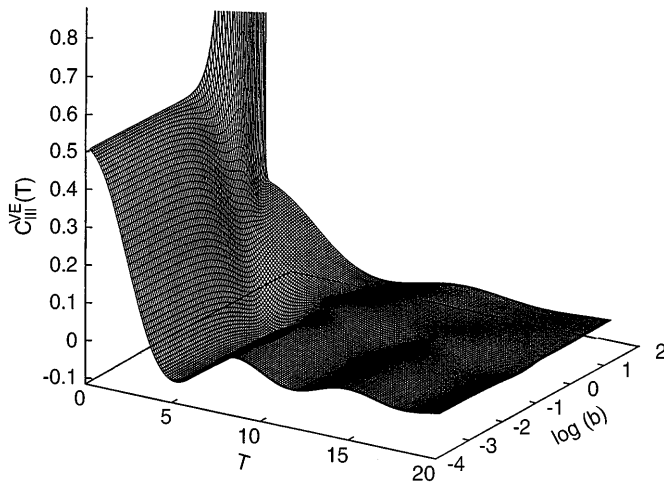


Fig. 5. Convolution kernel for the mode III viscoelastic problem for the SLS class of materials with $\zeta = 1$, showing a strong dependence on the mode number q (contained in the non-dimensional parameter $b = 1/|q|c_{so}\tau$)

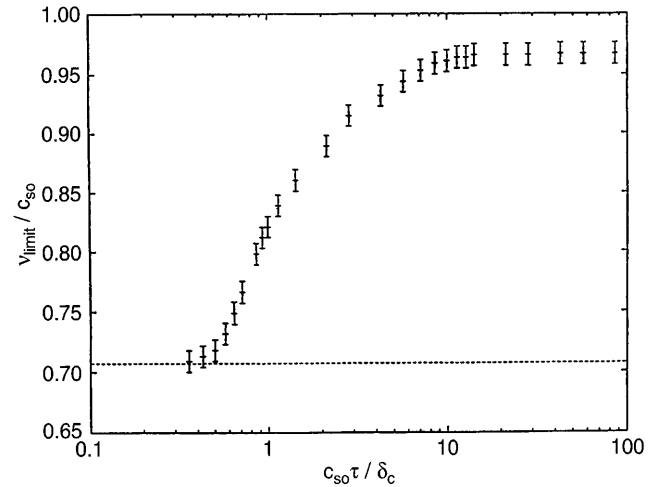


Fig. 6. Evolution of the limiting speed of a crack propagating in a SLS viscoelastic medium for various values of the relaxation time τ . The error bars are associated with the numerical computation of the crack velocity from the crack tip position

viscoelastic case, on the mode number $|q|$ through the non-dimensional parameter b .

Details on the numerical implementation, precision and stability of the viscoelastic spectral scheme can be found in Geubelle, Danyluk and Hilton (1996), together with a series of examples illustrating the damping effect of the surrounding viscoelastic medium on the crack response to impact-like loading conditions, and on the energy balance taking place in the vicinity of the tip of a rapidly propagating fault.

This latter issue is illustrated in Fig. 6, which presents the variation of the limiting velocity of a spontaneously propagating mode III crack with respect to the relaxation time τ . The cohesive model was chosen as the same simple rate-independent linear law as in the dynamic debonding problem shown in Fig. 3 to allow to focus primarily on the effect of the possible relaxation taking place in the surrounding medium. For large values of τ , the material does not have time to relax and the propagation takes place elastically, with a limiting velocity given by $c_{so} = \sqrt{\mu_o/\rho}$. As τ decreases, we observe a progressive transition to a lower limiting speed corresponding to the fully relaxed value $c_{s\infty} = \sqrt{\mu_{\infty}/\rho}$.

References

- Breitenfeld, M. S.; Geubelle, P. H. (1997): The mechanics of 2D and 3D dynamic interfacial fracture (in preparation)
 Cochard, A.; Rice, J. R. (1996): A spectral method for numerical elastodynamic fracture analysis without spatial repli-

cation of the rupture event. Division of Applied Sciences, Harvard University, Cambridge, MA (to appear in J. Mech. Phys. Solids)

Danyluk, M. J.; Geubelle, P. H.; Hilton, H. H. (1996): 3D dynamic fracture in viscoelastic media. AAE 96-13 Report, Dept. Aero. & Astro. Eng., Univ. Illinois Urbana-Champaign (submitted to Int. J. Sol. Struc.)

Geubelle, P. H. (1994): Implementation of a 3D elastodynamic boundary-integral code on the CM-5. Report Mech-240, Div. Appl. Sciences, Harvard University, Cambridge, MA

Geubelle, P. H.; Breitenfeld, M. S. (1996): Numerical analysis of dynamic debonding under anti-plane shear loading. AAE 96-12 Report, Dept. Aero. & Astro. Eng., Univ. Illinois Urbana-Champaign (submitted to Int. J. Frac.)

Geubelle, P. H.; Rice J. R. (1995): A spectral method for 3D elastodynamic fracture problems. J. Mech. Phys. Solids. 43 (11) 1791-1824

Geubelle, P. H.; Danyluk, M. J.; Hilton, H. H. (1996): Dynamic Mode III fracture in a viscoelastic medium. AAE 96-03 Report, Dept. Aero. & Astro. Eng., Univ. Illinois Urbana-Champaign. (to appear in Int. J. Sol. Struc.)

Morrissey, J. W.; Geubelle, P. H. (1995): A numerical scheme for mode III dynamic fracture problems. Int. J. Num. Meth. Eng. 40, 1181-1196

Perrin, G.; Rice, J. R.; Zheng, G. (1995): Self-healing slip pulse on a frictional surface. J. Mech. Phys. Solids. 43 (9) 1461-1495

Rice, J. R.; Ben-Zion, Y. (1996): Slip complexity in earthquake fault models. Proc. Nat. Acad. Sci. USA 93, 3811-3818

Yu, H.; Yang, W. (1994): Mechanics of transonic debonding of a bimaterial interface: the anti-plane case. J. Mech. Phys. Solids. 42, 1789-1802