AE 513 Multiphase Flow, HW #4
Due May 9, 2008 (Chapters 4 & 5)

For **derivations** show **all** steps and list **all** assumptions.

1) **4 points** Describe a key benefit (i.e. reason for choosing) for each of the following techniques:
   a) Lagrangian particle vs. Lagrangian parcel approaches,
   b) Lagrangian particle vs. Eulerian bin approaches for polydisperse particles,
   c) mixed-fluid vs. separated-fluid vs. weakly-separated approaches, and
   d) point-force vs. distributed force vs. resolved-surface approaches.

2) **4 points Derive** the two-way coupled continuous-phase momentum transport of Eq. 4.22a starting from the Reynolds Transport Theorem of Eq. A.1.

3) **4 points** a) **Derive** Eq. 4.46e. b) Develop a guide like Eqs. 4.46f or 4.46g for sand particles in water with $\rho_p=2.5\rho_f$.

4) **4 points** Choose a dispersed multiphase flow problem and flow aspects which you would like to predict. a) Based on Chapter 4, describe the equations and numerical approaches that would be appropriate and explain your reasoning for each. b) Estimate the number of continuous-fluid nodes and particle nodes which may be needed to simulate your flow.

5) **2 points** Starting from the separated-fluid Eulerian dispersed-phase and continuous-phase momentum equations (Eqs. 4.20b and 4.22a), **derive** the mixture momentum equation (Eq. 4.17b) for conditions of Eq. 4.15a.

6) **4 points** Based on Eq. 5.30, **derive** Eq. 5.37 for isentropic bubbles with negligible surface tension.

7) **6 points** Consider a 1-D mixed-fluid multiphase flow moving at a constant velocity of 5 μm/s which contains water and neutrally-buoyant spherical particles. **Derive** discretized equations for the Eulerian volume fraction transport of particles in this flow. Use these equations to solve for the unsteady volume fraction distribution in a domain of x=0,L where L=100 μm with the initial conditions: $\alpha=0$ for $x\leq 0.4L$, $\alpha=0.1$ for $x\geq 0.5L$ and α varies linearly for $0.4L\leq x\leq 0.5L$. Use a grid resolution of 100 cells and a sufficiently small temporal resolution to ensure time-step independence (and stability). Plot the volume fraction distribution for t = 0, 1, 2 and 4 seconds and qualitatively discuss the results.

8) **4 points** a) Starting from Eq. 5.47, **derive** the following mixed-fluid momentum equation:

$$\frac{\partial\left(\bar{\rho}_m\tilde{V}_{mi}\tilde{V}_{mj}\right)}{\partial x_j}=\bar{\rho}_m g_i-\frac{\partial \bar{p}}{\partial x_i}+\frac{\partial}{\partial x_i}\left[\left(\mu_m+\mu_{m,t}\right)\left(\frac{\partial \tilde{V}_{mi}}{\partial x_j}+\frac{\partial \tilde{V}_{mj}}{\partial x_i}\right)-\frac{2}{3}k_m\right]$$

b) Explain the physics-based motivation for using a Fickian diffusion to form Eq. 5.74b.

9) **3 points** a) Starting with Eq. 3.5, **derive** Eq. 5.83 and show that is second-order accurate with respect to τ_p. b) Starting with Eq. 5.87f, **derive** Eq. 5.87i.

10) **6 points** Consider a potential flow solution whose stream function is given by $\psi = u_\infty y\left[1 - r_o^2/\left(x^2 + y^2\right)\right]$ and a very heavy particle (R→0) with linear drag, a Stokes number defined as $St_D = \tau_p u_\infty / r_o$ and a terminal velocity given by $\mathbf{w}_{term,y} = -0.1 u_\infty \mathbf{i}_y$

a) Use the Eulerian weakly-separated approach of Eq. 5.83b to analytically obtain v_y/u_∞ at x/r_o=-1.1 for y/r_o ranging from -3 to +3 for St_D=0.2 and 2.
b) Use the terminal-velocity approach of Eq, 5.84 to similarly obtain v_y/u_∞.
c) Use the separated-fluid Lagrangian approach of Eq. 3.17 to obtain v_y/u_∞ for particles when they cross the plane at x/r_o=-1.1 but are initially released with a velocity equal to that of the fluid and at positions with y/r_o of -2, -1, 0, +1, +2, +3 and x/r_o=-2. Make sure your time-step is small enough that your results are time-accurate.
d) For St_D=0.2, plot v_y/u_∞ as a function of y/r_o for the three different techniques (use solid and dashed lines for the two Eulerian approaches and symbols for the discrete values of the separated-fluid approach). Make a similar plot for St_D=2. Discuss the differences among the three techniques for these two conditions.