My English is not so good to be able to make comments on your grammar or writing style. What I am doing here is just put in something that I think it could make the material easier to be understood. You might need to rewrite some of them because I am not sure I have written them in a Standard English style.

Since this is a textbook, but not a journal paper, I think more intermediate steps are needed. Also, definitions are more welcome to be written in formula format, and each formula should occupy a single line. Important key words should be bold or italicized.

1.2.3. Two-Way Coupling Criteria

Two-way coupling in two-phase flows generally denotes that inter-phase transfer of mass, momentum or energy is important to the fluid dynamic description of both phases. In many two-phase flow simulations, the continuous-fluid will affect the dispersed-phase but not vice-versa. This is often referred to as one-way coupling (and defined herein as dilute multiphase flow conditions), whereby the continuous fluid computation need not require a modified treatment as compared to computation of a single-phase flow. Such a simplification can represent a significant reduction in required computational resources and/or development, and therefore it is important to take advantage of this simplification if it is appropriate. As such the level of particle loading for which two-way coupling becomes important, also identifies the limit of applicability of the dilute conditions.

Crowe *et al.* (1998) gives criteria for the limits of inter-phase transfer of momentum, energy or mass to retain one-way coupling using certain assumptions. In this text, only the momentum-based two-way coupling will be considered ~~herein~~ and will be analyzed in two limits: 1) where the particle relative velocity is primarily derived from gravitational forces, and 2) where the particle relative velocity is primarily derived from rapid variations in the continuous-phase fluid. Some basic understanding of the particular multi-phase flow of interest is necessary to determine which criterion is applicable. In addition to considering these two flow regimes, momentum coupling (due to the presence of the dispersed-phase) will be considered both in terms of: a) changes in the net momentum of the continuous-phase, and b) changes in continuous-phase secondary features, such as the turbulence or spatial/temporal perturbations of the flow.

Gravitationally driven conditions

Comment: This kind of subtitles could help readers quickly locate the target materials

To considering whether two-way coupling exists, Crowe *et al.* (1998) defined a ~~momentum-coupling parameter~~ *momentum-coupling parameter* (Π) ~~as the ratio of the particle force imparted on the continuous-fluid to the momentum flux of the continuous-phase, in a unit volume of multiphase flow~~.

Comment: It is better to italicize or bold the keywords.

$$\Pi = \frac{\Sigma F_p}{momentum\_flux_c}$$

Comment: I think it is better to put such important definitions in a single line, and write them in a formula format.

where F_p represents the particle force imparted on the continuous fluid. One can assume that the average restitution force imparted from the particles is primarily based on the *drag force*

(D_p), and thus is related to the magnitude of the particle ~~kinematic~~ kinetic non-equilibrium, i.e. the *relative velocity* between the particle and the continuous-fluid (V_{rel}).

The most straight-forward evaluation of momentum coupling back to the continuous-fluid is the condition where the relative velocity is primarily controlled by *gravitational forces* (G_p), and on the order of the *particle terminal velocity* (V_{term}). In this condition, particles faithfully follow the continuous flow (St_L<1), and the stream-wise component of the relative velocity is negligible. Thus the continuous phase velocity plays little role on the drag forces. Let's assume that the drag forces are independent of V_{rel} and are equilibrated by the *relative gravity forces* (ΔG_p), then we have:

Comment: You might need to give a clear definition about the particle terminal velocity.

$$\left|F_p\right| \sim \left|D_p\right| \sim \left|\Delta G_p\right| = \left|\rho_p - \rho_f\right| g L^3 \alpha \qquad when \quad V_{rel} \sim V_{term}$$

Comment: I put some intermediate steps here.

where α is the *volume fraction* of the dispersed phase per unit volume of mixed fluid (and is much less than unity for dilute flow). In general, this will be true if the particle response time is smaller than the system time-scales, i.e. if St_L<1. Let us first consider the *macroscopic momentum-coupling parameter* for the system volume, L^3, under the conditions of dilute flow?

Comment: I moved this sentence from the next paragraph to here.

Comment: According to the definition in previous section, 'dilute' means only one-way coupling. So why you mention dilute flow here?

$$\Pi_L = [\alpha\, L^3\, |\rho_p - \rho_f|\, g] \,/\, [\rho_f\, V_L^2\, L^2] \;=\; \alpha\, |\Psi - 1| \,/\, Fr_L \qquad\qquad St_L<1$$

where Fr_L is the *macroscopic Froude number*:

$$Fr_L = V_L^2/gL$$

If the macroscopic coupling parameter is much less than unity, one may reasonably neglect the effect of particle restitution of the continuous-fluid bulk momentum, and therefore assume one-way coupling with respect to V_L. (In gravity-controlled conditions, although the relative velocity play no roles on the restitution forces, but it influences the coupling criteria through the continuous fluid flux, which is reflected in Froude number)

The simple relationship derived for Π_L indicates some important trends. As expected, the degree of loading is linearly proportional to the volume fraction of the particle phase within the mixture volume. For very heavy particles (Ψ>>1), the degree of loading is also linearly proportional to the density ratio. If one defines the particle mass per continuous-fluid mass for a mixed volume as the *particle mass loading* (η) (~~particle mass per continuous-fluid mass for a mixed volume~~):

$$\eta = \alpha\rho_p/\rho_f$$

then

$$\Pi_L \sim \eta/Fr_L \qquad\qquad \Psi>>1$$

For very-buoyant particles (Ψ<<1), the coupling will be independent of the density ratio and

$\Pi_L \sim \alpha / Fr_L$ $\Psi << 1$

In addition, one might expect that flows with very-buoyant particles ~~(or should we call these low-density particles?)~~ flows will be typically slower than flows the very-heavy particles ~~(or should we call these high-density particles?)~~ so that the Froude numbers for the latter are expected to be typically larger. While these dependencies are simply expected, the influence on Froude number is interesting. If one assumes that gravity is constant (and important to the particle relative velocity), the macroscopic coupling yields a quadratic decrease with mean fluid velocity but increases linearly with system size. As such, the coupling becomes more pronounced for slower and larger flow systems if particle density ratio and volume fraction are held fixed. What is also interesting about this parameter is that it is independent of particle size and Reynolds number.

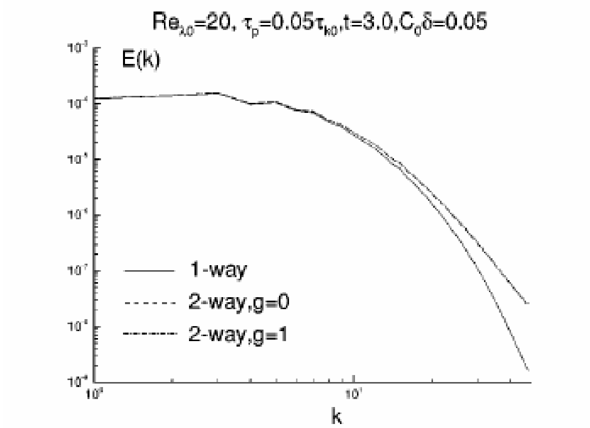
Comment: I think heavy and buoyant is OK. On one hand, you have given the definitions of heavy particles and buoyant particles in previous paragraph. On the other hand, you want to express an idea of relative density. So heavy and buoyant are better than high and low density.

Comment: Why we expect this? Is it because we need to keep St<1?

Now let us consider the (more sensitive) coupling criteria whereby secondary aspects of the continuous-fluid can be modified. For turbulent flow, Froude numbers based on the integral scales and Kolmogorov scales (i.e. $Fr_\Lambda = V_\Lambda^2/g\Lambda$ and $Fr_K = V_K^2/g\lambda_K$, where Λ is the Kolmogorov length scale and λ_K is the Kolmogorov length scale) similarly yield integral and Kolmogorov momentum coupling parameters ($\Pi_\Lambda = \alpha|\Psi - 1|/Fr_\Lambda$ and $\Pi_K = \alpha|\Psi - 1|/Fr_K$). These two momentum coupling parameters consider the momentum of the velocity fluctuations associated with the integral scales and the Kolmogorov scales, respectively. To obtain a general idea of how much more sensitive these criteria are to the macroscopic coupling parameter, one may estimate the ratio of Froude numbers for a simple free shear flow. In this case, one may roughly approximate:

$$\Lambda/L \sim V_\Lambda / V_L$$

where both ratios are typically less than 10^{-1}, such that Π_L is typically much less than Π_Λ, i.e. coupling to the turbulence would be at least an order of magnitude more important than coupling to the mean fluid. Similarly, Π_Λ is typically much less than Π_K for high Reynolds number flows. Similar coupling parameters can be defined based on secondary flow features, e.g. vortices, transverse fluid velocity variations, etc. by defining a Froude number associated with those features. As such, a multi-phase flow may have significant coupling on the secondary features but with negligible coupling on the mean flow features. An example of this is the turbulent pipe flow (at 10 m/s) with 70 micron copper spheres at a mass loading of 80% experimentally investigated by Kulick *et al.* (1994). The mean velocity profile was not strongly influenced by the particles' presence (as shown in Figure 9) which was consistent with a low mean flow coupling parameter ($\Pi_L \sim 0.01$); however, the turbulence distribution in the outer-layer was found to be significantly reduced (as shown in Figure 10) which was consistent with a sizable (outer-layer) integral-scale coupling parameter ($\Pi_\Lambda \sim 0.5$). Another example is the numerical study of isotropic turbulence by O. A. Druzhinin etc. (1999). In that study, the simulation was done by a one-way coupling model and a two-way coupling model respectively. The result shows owing to the two-way coupling, both the kinetic energy and the dissipation are increased *only* in high wave number regime (small flow structures) compared to the one-way coupling case (figure below).



Spectrum of the turbulence kinetic energy, $E(k)$, in the one-way and two-way coupling cases. τ_p is the particle respond time, τ_{k0} is the Kolmogorov time scale, $C_0\delta$ is the mass loading, g=0 means without gravity and g=1 means with gravity.

The three momentum-coupling criteria defined above can be summarized (under the assumption that V_{rel} can be approximated by V_{term}) as follows:

$\Pi_L << 1$ *for negligible two-way coupling on mean flow when* $V_{rel} \sim V_{term}$

$\Pi_\Lambda << 1$ *for negligible two-way coupling on turbulent energy when* $V_{rel} \sim V_{term}$

$\Pi_K << 1$ *for negligible two-way coupling on turbulent spectrum when* $V_{rel} \sim V_{term}$

Note, that the above coupling parameters indicate the capability of a multi-phase flow to incur two-way coupling, but do not specify whether the continuous-phase flow properties will be increased or decreased. However some qualitative predictions can be made. A terminal velocity in the direction of the bulk flow, will typically tend to increase V_Λ. With respect to turbulence, the presence of particles generally tends to increase the turbulent dissipation rate and therefore decrease the overall turbulence level (e.g. Kulick *et al.* 1994). However, this result can be different under two extremes of very large or very small particles. For particles with high relative velocities, the wakes of the particles provide additional unsteadiness or distortion to the flow, especially if the wakes are naturally turbulent; an example is the flow experiments by Parthesarathy & Faeth (1990) for particles with $V_{term}>>V_\Lambda$ and $Re_p>>1$. For the case of micro-particles, the particles yield an increased inertia of the fluid such that the dissipation (kinetic energy decay rate) is reduced as compared to the one-way coupled flow; an example is the flow simulation results of Druzhinin & Elghobashi (1999) with $S_K <<1$ and $\Pi_\Lambda \sim 1$.

Continuous-fluid driven conditions

The above criteria are for conditions where gravity primarily drives the particle relative velocity. Conditions where this may not be true include multi-phase flow with particles which are neutrally (or near neutrally) buoyant, such as immiscible droplets in a liquid or blood cells in a capillary vein. Micro-gravity flows (Groszman *et al.* 1999) are another obvious example where gravitational forces may be not primary. In addition, particle non-equilibrium can arise principally due to rapid variations in flow properties with respect to time or space, e.g. entrained dust can provide strong shock attenuation (Sivier *et al.* 1994); similarly droplet injection into an internal-combustion engine typically yields relative droplet velocities that are significantly in excess of the terminal velocity (Gosman & Ioannides, 1981). Similarly, the relative velocity may not be equal to the terminal velocity for any flow where $S_L>1$, or even where $S_\Lambda>1$ if eddy structures dominate the flow. In such cases, the relative velocity is typically determined by the initial particle velocity or by the change in the continuous-fluid velocity along the path of the particle. In the latter case (particles hardly follow the continuous flow and don't have a significant initial velocity), one may assume that the relative velocity is of the order of the system velocity (V_{rel}~V_L). Similarly assuming that the resulting net particle drag force is on the order of the particle relative momentum flux, i.e. :

$$D_p \sim momentum\_flux_{rel} = \alpha\left(\rho_p + C_M \rho_f\right) V_{rel}^{\,2} L^2 \sim \alpha\left(\rho_p + C_M \rho_f\right) V_L^{\,2} L^2$$

Comment: I think intermediate steps like this is necessary for a textbook.

and the macroscopic momentum coupling can be approximated:

$$\Pi_L = [\alpha\,(\rho_p + C_M\,\rho_f\,)\,V_L^2\,L^2\,] / [\rho_f\,V_L^2\,L^2] = \alpha\,(\Psi + C_M)$$

This indicates a significant importance on the particle to continuous-fluid density ratio (Ψ). As such, the primary non-dimensional parameter of influence for very-heavy particles is the mass loading, while it is the void fraction for very-buoyant particles. These two limits yield the following criteria

$\eta << 1$ *for negligible two-way coupling on mean flow when* V_{rel}~V_L *and* $\Psi>>1$

$C_M\,\alpha << 1$ *for negligible two-way coupling on mean flow when* V_{rel}~V_L *and* $\Psi<<1$