1/24/08 Satellite of the Day! GPS IIF

- state-of-the-art GPS satellite, Boeing, 2008
- Feb 1978 GPS first launched
- GPS constellation can hold up to 30, currently ~25
- need 4 satellites, 3 from satellites to determine position, 1 for adjusting for time bias
- ~20,000 km altitude, 2 rotations about Earth per day
- 3-axis active control

Euler Sequences

Asymmetric: 123, 132, 213, 231, 312, 321

Symmetric: 121, 131, 212, 232, 313, 323

Ω, i, ω — "classical euler angle sequence"

$$R_{313}(\phi,\theta,\psi) = R_3(\psi) R_1(\theta) R_3(\phi)$$

$$R_3(\psi)=\begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix},\quad R_1(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix},\quad R_3(\phi)=\begin{bmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$R_{313}=\begin{bmatrix}c_\phi c_\psi - c_\theta s_\phi s_\psi & c_\psi s_\phi + c_\theta c_\phi s_\psi & s_\theta s_\psi\\ -c_\theta c_\psi s_\phi - c_\phi s_\psi & c_\theta c_\phi c_\psi - s_\phi s_\psi & c_\psi s_\theta\\ s_\theta s_\phi & -c_\phi s_\theta & c_\theta\end{bmatrix}$$

$R_{313}(\phi,\theta,\psi) = R_{313}(\phi+\pi, -\theta, \psi-\pi)$ => not unique rotions to angles

Constrain the angles to give uniqueness

domain of angles $0 \le \phi < 2\pi$

$0 \le \theta < \pi$

$0 \le \psi < 2\pi$

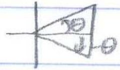
$R_{ijk}(\phi,\theta,\psi) = R_{ijk}(\phi+\pi, \pi-\theta, \psi-\pi)$

constrain: $0 \le \phi \le 2\pi$

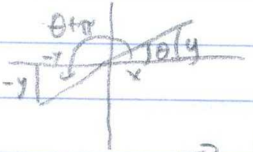
$-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$

$0 \le \psi < 2\pi$

$$R_{3,3} = \begin{bmatrix} [R_{3,3}]_{1,1} & [R_{3,3}]_{1,2} & [R_{3,3}]_{1,3} \\ [R_{3,3}]_{2,1} & [R_{3,3}]_{2,2} & [R_{3,3}]_{2,3} \\ [R_{3,3}]_{3,1} & [R_{3,3}]_{3,2} & [R_{3,3}]_{3,3} \end{bmatrix}$$

$\theta = \pm\cos^{-1}([R_{3,3}]_{3,3})$ 

$$\left.\begin{aligned} S_\theta S_\phi &= [R_{3,3}]_{3,1} \\ -S_\theta C_\phi &= [R_{3,3}]_{3,2} \end{aligned}\right\} \quad -\tan\phi = \frac{[R_{3,3}]_{3,1}/\cancel{\sin\theta}}{[R_{3,3}]_{3,2}/\cancel{\sin\theta}}$$



$\tan\theta = \frac{y}{x}$

$\tan(\theta+\pi) = \frac{-y}{-x}$

$\longrightarrow \theta = \mathrm{atan2}(y, x)$

(restrict domain to eliminate problem for MATLAB

$$\phi = \mathrm{atan2}(\overset{y}{[R_{3,3}]_{3,1}}, \overset{x}{[R_{3,3}]_{3,2}})$$

$$\theta = \cos^{-1}([R_{3,3}]_{3,3})$$

$$\psi = \mathrm{atan2}([R_{3,3}]_{1,3}, [R_{3,3}]_{2,3})$$