Today's Satellite: Galileo. 5/2/08

Mission: to Jupiter.

ADCS: dual-spin satellite. Main dish is "de-spun" with respect to the rest of the system.

- How do we get electrical signals across the spin plane?
- High gain antenna failed → had to use low gain antenna for all data.

All rotation matrices have an eigenvalue of 1 for which the eigenvector is the axis of rotation.

$$R v = \cancel{\lambda} v'$$

$$R^{B/A} v_A = v_B$$

Note: the rotation axis will have the same coordinates in both the original and new frames of reference.

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$\hat{b}_1$ in $\hat{a}$ frame will be the top row.
$\hat{a}_1$ in $\hat{b}$ frame will be the left column.

Advantages of using a Rotation Matrix:
→ no ambiguities
→ easy to use.

Disadvantages

→ need to store 9 numbers instead of a sequence and 3 angles

→ need to verify that it is a rotation matrix.

$$\hat{a}_1 = R_{11}\hat{b}_1 + R_{21}\hat{b}_2 + R_{31}\hat{b}_3$$

$$\frac{{}^{A}d\hat{a}_1}{dt} = 0 \quad \text{since } a \text{ is inertial.}$$

$$\frac{{}^{A}d\hat{a}_1}{dt} = \left[(\dot{R}_{11}\hat{b}_1) + {}^{A}\vec{\omega}^{B} \times R_{11}\hat{b}_1\right] + \left[(\dot{R}_{21}\hat{b}_2 + {}^{A}\vec{\omega}^{B} \times R_{21}\hat{b}_2\right]$$
$$+ \left[\dot{R}_{31}\hat{b}_2 + {}^{A}\vec{\omega}^{B} \times R_{31}\hat{b}_3\right]$$

where: ${}^{A}\vec{\omega}^{B} = \omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3$

$$0 = (\dot{R}_{11}\hat{b}_1 + \dot{R}_{21}\hat{b}_2 + \dot{R}_{31}\hat{b}_3) + R_{11}(-\omega_2\hat{b}_3 + \omega_3\hat{b}_2)$$
$$+ R_{21}(\omega_1\hat{b}_3 - \omega_3\hat{b}_1) + R_{31}(-\omega_1\hat{b}_2 + \omega_2\hat{b}_1)$$

$$\therefore \dot{R}_{11} - R_{21}\omega_3 + R_{31}\omega_2 = 0$$

$$\boxed{\dot{R}_{11} = \omega_3 R_{21} - \omega_2 R_{31}}$$

$$\dot{R}^{B/A} = -S({}^{A}\vec{\omega}^{B})\, R^{B/A}$$

$$S({}^{A}\vec{\omega}^{B}) \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad \text{Skewed Symmetric.}$$

See handout.

A complex number represents a rotation.

which leads to a quaternion representation:

$$i q_1 + j q_2 + k q_3 + q_4 \qquad \text{where} \quad i^2 = j^2 = k^2 = -1$$

$$ij = k, \quad ji = -k$$