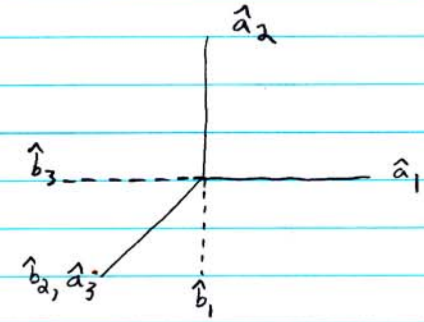
AE 403 2/12/08




Find: (0) direction cosine matrix
(1) the quaternion
(2) equivelant axis angle

(0)
$$R^{B/A} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

(1)
$$q_1 = \pm \frac{1}{2} = \frac{1}{2}$$

$$q_4 = \frac{1}{4q_1}(1-0) = \frac{1}{2}$$ $\longrightarrow$ { $q_4$ is positive by convention so choose positive $q_1$ }

$$q_2 = \frac{1}{2}(-1) = -\frac{1}{2}$$

$$q_3 = \frac{1}{2}(-1) = -\frac{1}{2}$$

$$Q = i\left(\tfrac{1}{2}\right) - j\left(\tfrac{1}{2}\right) - k\left(\tfrac{1}{2}\right) + \tfrac{1}{2} = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}$$

$$q = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad q_4 = \frac{1}{2}$$

or
$$q_2 = \pm\frac{1}{2} = -1/2, \quad q_4 = \frac{1}{4q_2}(-1-0) = 1/2$$

$$q_1 = -\frac{1}{2}(-1) = \frac{1}{2}, \quad q_3 = -\frac{1}{2}(1) = -\frac{1}{2}$$

(2) $\mu = 2\cos^{-1}(q_4) = 120°$

$$\hat{e} = \frac{q_1 \hat{a}_1}{\sqrt{q_1^2 + q_2^2 + q_3^2}} + \ldots = (1/\sqrt{3})\hat{a}_1 - (1/\sqrt{3})\hat{a}_2 - (1/\sqrt{3})\hat{a}_3$$

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Rotate 90° about $\hat{b}_1$ then 90° about $\hat{b}_2$, Find the resulting quaternions.

$e_1 = 1$ $\quad \mu = 90°$ $\quad P = i\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}$

$e_2 = 0$ $\quad Q = j\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}$

$e_3 = 0$

$$(PQ) = \left(i\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\left(j\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$= \frac{1}{2}k + \frac{1}{2}i + \frac{1}{2}j + \frac{1}{2}$$

$$= \frac{1}{2}(i + j + k + 1)$$

$$= Q \otimes P \quad , \quad P = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}\frac{1}{\sqrt{2}} \qquad Q = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}\frac{1}{\sqrt{2}}$$

$$\Rightarrow Q \otimes P = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$