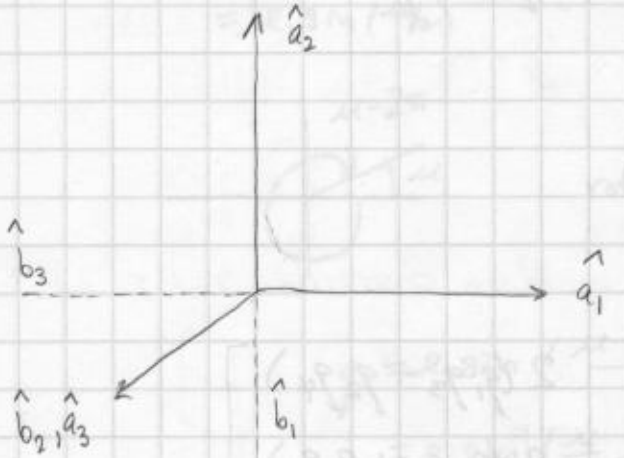
2/12/2008

Satellite of the day - ISS Control moment gyro



Find ⓐ DCM
① quaternion
② the eq axis/angle

$$A\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix}_b \qquad R^{B/A} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$q_1 = \pm\frac{1}{2}\sqrt{1} = \pm\frac{1}{2}$$

$$q_4 = \frac{1}{4q_1}(1-0) = \frac{1}{2}$$

$$q_2 = \frac{1}{4(\frac{1}{2})}(0-1) = -1/2$$

$$q_3 = \frac{1}{4(\frac{1}{2})}(0-1) = -\frac{1}{2}$$

$$Q = i\tfrac{1}{2} - j\tfrac{1}{2} - k\tfrac{1}{2} + \tfrac{1}{2}$$

$$= \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}, \quad q = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \end{bmatrix} \quad q_4 = 1/2$$

using a different method

$q_2 = \pm 1/2 \Longrightarrow -\frac{1}{2}$

$q_4 = \frac{1}{4q_2}(-1-0) = 1/2$

$q_3 = \frac{1}{4(-1/2)}(1+0) = -1/2$

$q_1 = \frac{1}{4(-1/2)}(-1+0) = 1/2$

same as before

(2) $q_1 = e_1 \sin(u/2)$

$q_2 = e_2 \sin(u/2)$

$q_3 = e_3 \sin(u/2)$

$q_4 = \cos(u/2)$

$u = 2\cos^{-1}(q_4) = 120°$

$\hat{e} = \frac{q_1}{\sqrt{q_1^2+q_2^2+q_3^2}}\hat{a}_1 + \ldots$

$\hat{e} = \frac{1}{\sqrt{3}}\hat{a}_1 - \frac{1}{\sqrt{3}}\hat{a}_2 - \frac{1}{\sqrt{3}}\hat{a}_3$

Rotate 90° about $\hat{b}_1$, = P
then 90° about $\hat{b}_2$, = Q
Find the resulting quaternion

$e_1 = 1$

$e_2 = 0$

$e_3 = 0$

$u = 90°$

$\hat{e} = 1\hat{b}_1 + 0\hat{b}_2 + 0\hat{b}_3$

$P = i\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$Q = j\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$$(PQ) = \frac{1}{\sqrt{2}}(i+1)\frac{1}{\sqrt{2}}(j+1)$$

$$= \frac{1}{2}(ij+i+j+1)$$

$$= \frac{1}{2}(k+i+j+1)$$

side note

$$(i+k)(j+1) = (ij+i+kj+k)$$

$$(PQ) = Q \otimes P$$

See more about quaternions notes

$$Q \otimes P = \left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \frac{1}{2}$$