AE 403 2/14/08

$$\dot{Q} = \frac{1}{2}\Omega \otimes Q = \frac{1}{2}\begin{bmatrix} 0 & w_3 & -w_2 & w_1 \\ -w_3 & 0 & w_1 & w_2 \\ w_2 & -w_1 & 0 & w_3 \\ -w_1 & -w_2 & -w_3 & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$w_1 = w_3 = 2, \quad w_2 = 0$

$$\dot{Q} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}}_{A} Q$$

$$Q(t) = e^{At} Q(0)$$

$$\begin{aligned} \dot{q}_1 &= q_2 + q_4 \rightarrow \\ \dot{q}_2 &= -q_1 + q_3 \rightarrow \\ \dot{q}_3 &= -q_2 + q_4 \rightarrow \\ \dot{q}_4 &= -q_1 - q_3 \rightarrow \end{aligned} \boxed{\begin{aligned} sQ_1 - q_1(0) &= Q_2 + Q_4 \\ sQ_2 - q_2(0) &= -Q_1 + Q_3 \\ sQ_3 - q_3(0) &= -Q_2 + Q_4 \\ sQ_4 - q_4(0) &= -Q_1 - Q_3 \end{aligned}}$$

$$Q = (\quad) q_1(0)$$

$$QT = \mathcal{L}^{-1}((sI - A)^{-1})\, Q(0)$$

Rigid Body Dynamics (rotational)

$$\vec{F} = \frac{d^A \vec{L}}{dt}$$

$$\vec{M} = \frac{d^A \vec{H}}{dt}$$

$$\frac{d^A}{dt}(\vec{r}_c + \vec{r})$$

$$\vec{L} = \int {}^A\vec{v}\, dm$$
$$= m\, {}^A\vec{v}_c$$

$$\vec{H}_c = \int \vec{r} \times {}^A\vec{v}\, dm$$

$${}^A\vec{v} = {}^B\vec{v} + {}^A\vec{\omega}^B \times \vec{r}$$

$$\vec{H}_c = \int \vec{r} \times {}^A\vec{v}\, dm + \int \vec{r} \times ({}^A\vec{\omega}^B \times \vec{r})\, dm$$

$${}^A\vec{\omega}^B = w_1 \hat{b}_1 + w_2 \hat{b}_2 + w_3 \hat{b}_3$$

$$\vec{r} = r_1 \hat{b}_1 + r_2 \hat{b}_2 + r_3 \hat{b}_3$$