Thursday 2/14/2008

Satellite of the day - Explorer 1

spin stabilized but spin axis changed due to antenna dissipating energy

$$\dot{Q} = \frac{1}{2}\Omega \otimes Q$$

$$= \frac{1}{2}\begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}$$

$$\omega_1 = \omega_3 = 2 \quad , \quad \omega_2 = 0$$

$$\dot{Q} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}}_{A} Q$$

$$Q(t) = e^{At} Q(0) \qquad \dot{x} = Ax$$

$$\dot{q}_1 = q_2 + q_4 \longrightarrow s\,Q_1 - q_1(0) = Q_2 + Q_4$$
$$\dot{q}_2 = -q_1 + q_3 \longrightarrow s\,Q_2 - q_2(0) = -Q_1 + Q_3$$
$$\dot{q}_3 = -q_2 + q_4 \longrightarrow s\,Q_3 - q_3(0) = -Q_2 + Q_4$$
$$\dot{q}_4 = -q_1 - q_3 \longrightarrow s\,Q_4 - q_4(0) = -Q_1 - Q_3$$

$$Q = (\qquad)\, q_1(0)$$

solve system of equations above then can solve for q as a function of time

$$Q(t) = \mathcal{L}^{-1}\left((sI - A)^{-1}\right) Q(0)$$

$$\vec{F} = \frac{{}^{A}d\vec{L}}{dt} = m\,{}^{A}\vec{a}_c \qquad \vec{M}_c = \frac{{}^{A}d\vec{H}_c}{dt}$$

$$\vec{L} = \int {}^{A}\vec{v}\, dm \qquad \frac{{}^{A}d}{dt}(\vec{r}_c + r)$$

$$\vec{H}_c = \int \vec{r} \times {}^{A}\vec{v}\, dm$$

$${}^{A}\vec{v} = {}^{B}\vec{v} + {}^{A}\vec{\omega}^{B} \times r$$

$$\vec{H}_c = \underbrace{\cancel{\int \vec{r} \times {}^{B}v\, dm}}_{0} + \underbrace{\int r \times ({}^{A}\vec{\omega}^{B} \times \vec{r})\, dm}$$

$${}^{A}\vec{\omega}^{B} = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_3 \hat{b}_3$$
$$\vec{r} = r_1 \hat{b}_1 + r_2 \hat{b}_2 + r_3 \hat{b}_3$$

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$J_{11} = \int (r_2^2 + r_3^2)\, dm \quad \leftarrow \text{moment of inertia}$$

etc.

$J_{12} = J_{21} = -\int r_1 r_2 \, dm$ ← product of inertia

etc.

eigenvalues non zero
Principle moments of Inertia
all positive

$$J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \leftarrow \hat{b}_1, \hat{b}_2, \hat{b}_3$$

are principal axes

If $J_{12} = J_{21} = \dots = 0$

then J_{11}, J_{22}, J_{33} are <u>principal</u> m.o.i

minor - smallest moi

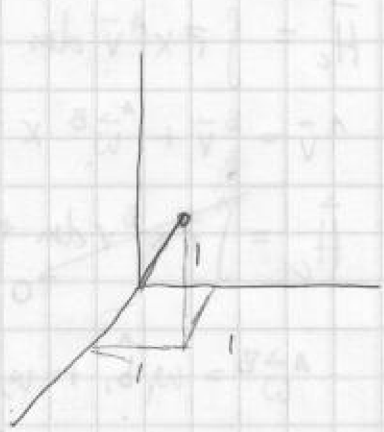
intermediate

major - largest m.o.i

$$\omega = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{H} = J\vec{\omega} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$${}^{A}\vec{\omega}^{B} = \hat{b}_1 + \hat{b}_2 + \hat{b}_3$$

$$\vec{H} = \hat{b}_1 + 2\hat{b}_2 + 3\hat{b}_3$$



Co-linearity

dot product = 1

cross product = 0

should scale

have equivalent unit vectors

$$\boxed{J\omega = \lambda\omega}$$