Satellite: Similar to TRANSIT, gravity-gradient stability, launched in late 80s

Polar BEAR: Beacon Experiment and Auroral Research

3x1 * coordinates always in frame B

$$H = J\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\lambda\omega = J\omega$$

λ: Eigenvalue of J = principal MOI

ω: Eigenvector of J = principal axes

Principal MOI: $\det(\lambda I - J) = 0$, Principal axes: $(\lambda_i I - J)e_i = 0$

Handout Example:

$$J = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Find principal MOI + principal axis for minor MOI

$\lambda = 1, 2, 3$

$$(\lambda_1 I - J)e_1 = 0 \Rightarrow \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-e_{11} - e_{12} = 0$, $-e_{11} - e_{12} = 0$, $-e_{13} = 0$

$\Rightarrow e_{11} = -e_{12}$, $e_{13} = 0$ $\Rightarrow \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$$\lambda_1 e_1 = 1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad Je_1 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \checkmark$$

$$e_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \qquad \text{for convention } e_1 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

$\vec{\omega} \times \vec{r} \leftrightarrow S(\omega)r$

$$S(e_1)e_2 = \begin{bmatrix} 0 & -e_{13} & e_{12} \\ e_{13} & 0 & -e_{11} \\ -e_{12} & e_{11} & 0 \end{bmatrix}\begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix} = -e_3$$

* want to end up with a coord. system that's right handed: take $-e_1$

Frame E: lined up with principal axes

$$R^{E/B} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$