2/21/08
Day 12

AE 403

$\dot{\omega} = J^{-1}(M - S(\omega)J\omega)$ $\dot{Q} = \frac{1}{2}\Omega \otimes Q$

$\vec{L} = m\,{}^A\vec{v}_c$ $T_{translation} = \frac{1}{2}m\,{}^A\vec{v}_c \cdot {}^A\vec{v}_c = \frac{1}{2}m\left|{}^A\vec{v}_c\right| = \frac{1}{2}\,{}^A\vec{v}_c \cdot (m\,{}^A\vec{v}_c) = \frac{1}{2}\,{}^A\vec{v}_c \cdot (\vec{L})$

$T_{rotation} = \frac{1}{2}\,{}^A\omega^B \cdot (\vec{H}) = \frac{1}{2}\omega^T J\omega$

$J^{-1}\dot{\omega} = M - S(\omega)J\omega$ assume body fixed frame is principal frame so $J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$

Solving for M gives us these three equations.

$M_1 = J_1\dot{\omega}_1 - (J_2 - J_3)\omega_2\omega_3$
$M_2 = J_2\dot{\omega}_2 - (J_3 - J_1)\omega_3\omega_1$
$M_3 = J_3\dot{\omega}_3 - (J_1 - J_2)\omega_1\omega_2$

Assume Torque free- $M_1 = M_2 = M_3 = 0$
Assume axisymmetric $J_1 = J_2 = J_t$ and $J_3 = J_a$
$0 = J_t\dot{\omega}_1 - (J_t - J_a)\omega_1\omega_2$
$0 = J_t\dot{\omega}_2 - (J_a - J_t)\omega_3\omega_1$
$0 = J_a\dot{\omega}_3 - (J_t - J_t)\omega_1\omega_2 = J_a\dot{\omega}_3$
The third equation yields that $\dot{\omega}_3 = 0$ or $\omega_3 = \text{constant}$
Let $\omega_3 = n = spinrate$
This yields $0 = J_t\dot{\omega}_1 - (J_t - J_a)n\omega_2$
$0 = J_t\dot{\omega}_2 + (J_t - J_a)n\omega_1$
Let $\lambda = \left(\frac{J_t - J_a}{J_a}\right)n$ the relative spin rate
This yields $0 = \dot{\omega}_1 - \lambda\omega_2$
$0 = \dot{\omega}_2 - \lambda\omega_1$
Multiply the first equation by ω_1 and the second by ω_2 yielding
$0 = \omega_1\dot{\omega}_1 - \lambda\omega_1\omega_2$ and $0 = \omega_2\dot{\omega}_2 + \lambda\omega_1\omega_2$
Adding the two equations yields $0 = \omega_1\dot{\omega}_1 + \dot{\omega}_2\omega_2$
This means $\omega_1^2 + \omega_2^2 = \text{constant}$
Note $\omega_1^2 + \omega_2^2 = \omega_t^2$ and $\left|{}^A\omega^B\right| = \omega_t^2 + n^2$

Remember that $T_{rotation} = \frac{1}{2}\omega^T J \omega$

$$T = \frac{1}{2}\begin{bmatrix}\omega_1 & \omega_2 & \omega_3\end{bmatrix}\begin{bmatrix}J_t & 0 & 0\\ 0 & J_t & 0\\ 0 & 0 & J_a\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}$$

$$T = \frac{1}{2}(J_t\omega_1^2 + J_t\omega_2^2 + J_a n^2) = \frac{1}{2}(J_t\omega_t^2 + J_a n^2)$$

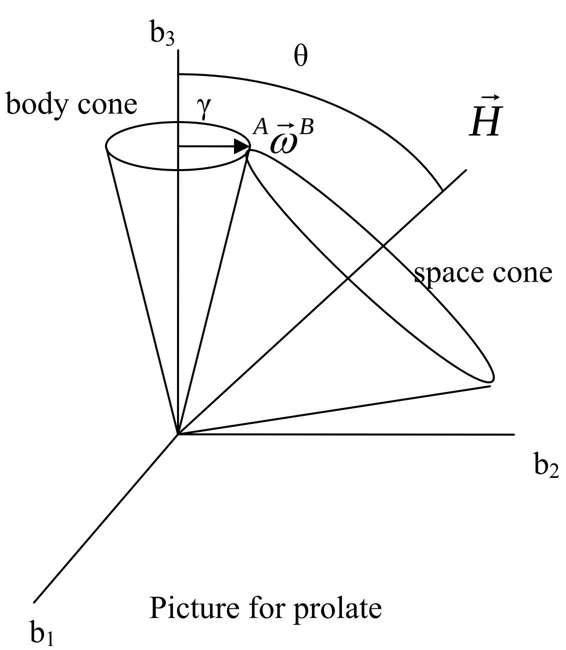
$\tan\gamma = \frac{\omega_t}{n}$ where γ is called the "wobble angle"

Note $H = \begin{bmatrix}J_t\omega_1\\ J_t\omega_2\\ J_a\omega_3\end{bmatrix}$ and $H_t = \sqrt{H_1^2 + H_2^2} = \sqrt{J_t^2(\omega_1^2 + \omega_2^2)} = J_t\omega_t$ and $H_a = J_a n$

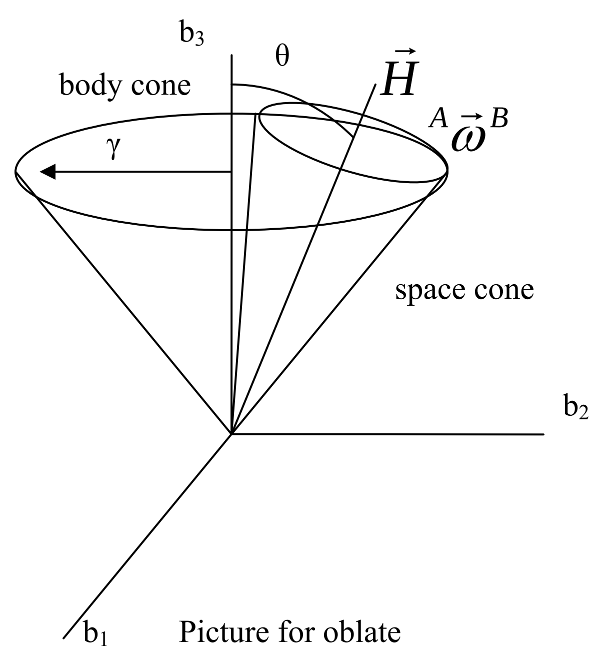
$\tan\theta = \frac{H_t}{H_a} = \frac{J_t\omega_t}{J_a\omega_a}$ where θ is the nutation angle

If $\tan\theta > \tan\gamma$ then $\frac{J_t\omega_t}{J_a n} > \frac{\omega_t}{n} \longrightarrow \frac{J_t}{J_a} > 1 \longrightarrow J_t > J_a$ this is a soup can shape or prolate and the motion is called direct procession

If $J_a > J_t$ that is a tuna can shape or oblate



Picture for prolate



Picture for oblate