2-28-08

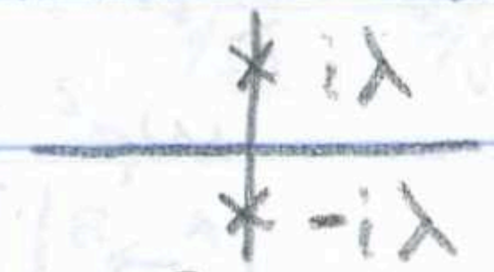
Satellite: DAWN (launched last Sept.)

- will investigate proto-planet asteroids
- will use yo-yo despin technique (46 rpm to 3 rpm in 4 sec)

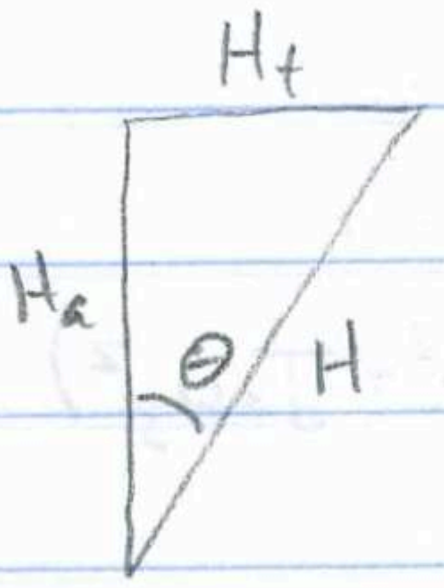
$$T = \frac{1}{2}\left(J_{11}\omega_1^2 + J_{22}\omega_2^2 + J_{33}\omega_3^2\right) = \frac{1}{2}\left(J_t\omega_t^2 + J_a\omega_3^2\right)$$

When we say spin is stable about major + minor axis: neutrally stable

$\Rightarrow$ poles are on imaginary axis



$$s^2 + \lambda^2 = 0 \Rightarrow s = \pm i\lambda$$



$$\sin^2\theta = \left(\frac{H_t}{H}\right)^2 = \frac{J_t^2\omega_t^2}{H^2} \qquad \cos\theta = \left(\frac{H_a}{H}\right)^2 = \frac{J_a^2\omega_3^2}{H^2}$$

$$\Rightarrow T = \frac{H^2}{2}\left(\frac{J_t^2\omega_t^2}{J_t H^2} + \frac{J_a^2\omega_3^2}{J_a H^2}\right)$$

$$T = \frac{H^2}{2}\left(\frac{\sin^2\theta}{J_t} + \frac{\cos^2\theta}{J_a}\right) = \frac{H^2}{2J_a}\left(\frac{J_a}{J_t}\sin^2\theta + 1 - \sin^2\theta\right)$$

$$= \frac{H^2}{2J_a}\left(1 + \sin^2\theta\left(\frac{J_a}{J_t} - 1\right)\right)$$

$$= \frac{H^2}{2J_a}\left(1 + \sin^2\theta\left(\frac{J_a - J_t}{J_t}\right)\right)$$

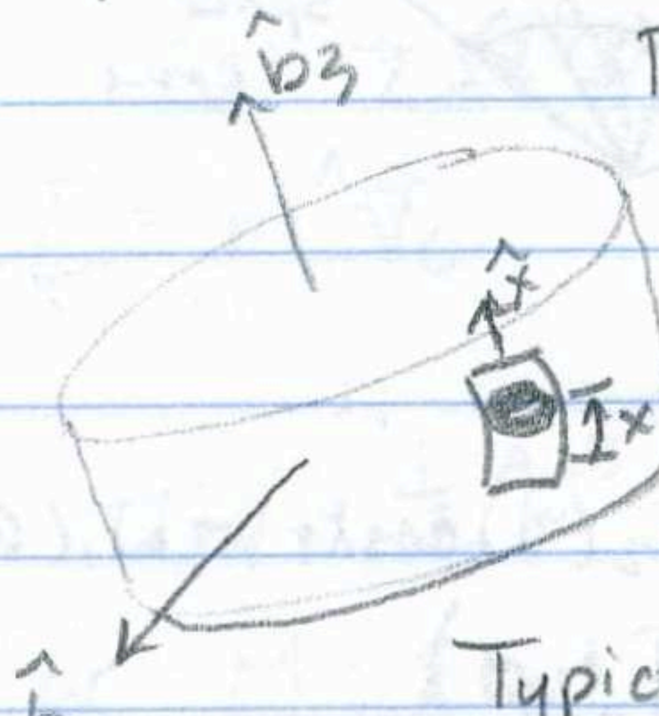
$$\dot{T} = \frac{H^2}{2J_a}\left(2\sin\theta\cos\theta\,\dot{\theta}\left(\frac{J_a - J_t}{J_t}\right)\right) = \frac{H^2}{J_a}\left(\sin\theta\cos\theta\,\dot{\theta}\left(\frac{J_a - J_t}{J_t}\right)\right)$$

What if $\dot{T} < 0$?

$$\dot{\theta}(J_a - J_t) < 0$$

$\Rightarrow$ prolate: $\dot{\theta} > 0$, oblate: $\dot{\theta} < 0$

Ball-in-Tube: for oblate spacecraft



$$\vec{f} = k\dot{x}\hat{x}$$

$$dW = -k\dot{x}\,dx = -k\dot{x}^2\,dt$$

$$W_{av} = \frac{1}{P}\int_0^P -k\dot{x}^2\,dt = -\frac{k}{P}\int_0^P \dot{x}^2\,dt$$

Typical model: $W_{av} = -c\theta^2$

$$\dot{T} = W_{av}$$

$$\Rightarrow \frac{H^2}{J_a}\left(\frac{J_a - J_t}{J_t}\right)\sin\theta\cos\theta\,\dot{\theta} = -c\theta^2$$

$$\frac{H^2}{J_a}\left(\frac{J_a - J_t}{J_t}\right)\theta\dot{\theta} = -c\theta^2 \quad \text{for small } \theta$$

$$\Rightarrow \dot{\theta} + \frac{1}{\tau}\theta = 0 \quad \text{where} \quad \tau = \frac{H^2}{J_a}\left(\frac{J_a - J_t}{J_t}\right)\frac{1}{c}$$

solution

$$\theta(t) = \theta_0 e^{-t/\tau}$$

$$J_a\omega_3(\text{final}) = \sqrt{J_t^2\omega_t^2 + J_a^2\omega_3^2}$$

$$\Rightarrow \omega_3(\text{final}) = \sqrt{\left(\frac{J_t}{J_a}\right)^2 + \omega_3^2} \quad \text{(final ang. vel. goes up)}$$

If: $J_a \omega_3 = J_t \omega_t \Rightarrow \omega_t = \frac{J_a}{J_t}\omega_3$

Applying torques about $\hat{b}_1 + \hat{b}_2$:

$$M_1 = J_t \dot{\omega}_1 - (J_t - J_a)\omega_2 \omega_3$$

$$M_2 = J_t \dot{\omega}_2 + (J_t - J_a)\omega_1 \omega_3$$

$$0 = J_a \dot{\omega}_3 \Rightarrow \omega_3(t) = n = \text{const}$$

$$\frac{M_1}{J_t} = \dot{\omega}_1 - \left(\frac{J_t - J_a}{J_t}\right) n \omega_2 \quad , \quad \frac{M_2}{J_t} = \dot{\omega}_2 + \left(\frac{J_t - J_a}{J_t}\right) n \omega_1$$

$$\Rightarrow \frac{M_1}{J_t} = \dot{\omega}_1 - \lambda \omega_2 \quad , \quad \frac{M_2}{J_t} = \dot{\omega}_2 + \lambda \omega_1$$

fix = 0

pulse