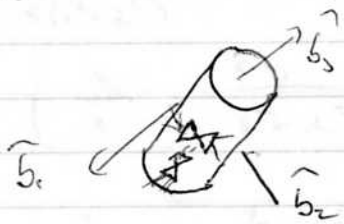
$0 = J_t \dot{\omega}_1 - (J_t - J_a)\omega_2\omega_3 = M_1$

$0 = J_t \dot{\omega}_2 + (J_t - J_a)\omega_1\omega_3 = M_2$

$0 = J_a \dot{\omega}_3$

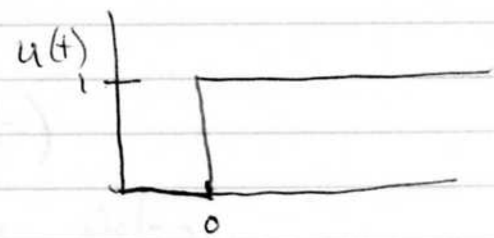
$\omega_3 = const = n$



$\frac{M_1}{J_t} = \dot{\omega}_1 - \lambda\omega_2 \qquad M_1 = 0$

$\frac{M_2}{J_t} = \dot{\omega}_2 + \lambda\omega_1 \qquad M_2 = M(u(t-\tau) - u(t-(\tau+\delta)))$

u = step function



Take a LaPlace Transform

$\mathcal{L}(u(t)) = \frac{1}{s}$

$\mathcal{L}(u(t-\tau)) = \frac{1}{s}e^{-s\tau}$

$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$

$\mathcal{L}(f(t-\tau)) = \int_0^\infty e^{-st} f(t-\tau)\,dt$

$y = t-\tau \qquad dy = dt$

$\mathcal{L}(f(t-\tau)) = \int e^{-s(y+\tau)} f(y)\,dy$

$= e^{-s\tau}\int_{0-\tau}^{\infty} e^{-sy} f(y)\,dy$

With $M_1 = M_2 = 0$

$\omega_1(t) = \omega_1(0)\cos\lambda t + \omega_2(0)\sin\lambda t$

$\omega_2(t) = \omega_2(0)\cos\lambda t + \omega_1(0)\sin\lambda t$

$\dot{y} + ky = u$

$u = \alpha_1 u_1 + \alpha_2 u_2$

$y = \alpha_1 y_1 + \alpha_2 y_2$

With an $M_2 = M u(t)$

$0 = s\omega_1 - \lambda\omega_2$

$\frac{M}{J_t}\left(\frac{1}{s}\right) = s\omega_2 + \lambda\omega_1$

$\omega_2 = \frac{1}{s}\left(-\lambda\omega_1 + \frac{M}{J_t}\frac{1}{s}\right)$

$0 = s\omega_1 - \frac{\lambda}{s}\left(-\lambda\omega_1 + \frac{M}{J_t}\frac{1}{s}\right)$

$$0 = s^2 w_1 + \lambda^2 w_1 - \frac{\lambda M}{J_t}\left(\frac{1}{s}\right)$$

$$w_1 = \lambda \frac{M}{J_t}\left(\frac{1}{s(s^2+\lambda^2)}\right)$$

$$w_1 = \frac{\lambda M}{J_t}\left(\frac{a}{s} + \frac{sb}{s^2+\lambda^2}\right)$$

$$as^2 + a\lambda^2 + bs^2 = 1$$

$$as^2 + bs^2 = 0 \qquad a = -b$$

$$a\lambda^2 = 1 \qquad a = \frac{1}{\lambda^2}$$

$$w_1 = \frac{M}{\lambda J_t}\left(\frac{1}{s} - \frac{s}{s^2+\lambda^2}\right)$$

$$w_1(t) = \frac{M}{\lambda J_t}\left(u(t) - \cos\lambda t\right)$$

To account for time delay

$$w_1(t) = w_1(0)\cos\lambda t + w_2(0)\sin\lambda t + \frac{M}{\lambda J_t}\left(u(t-\tau) - \cos\lambda(t\right.$$

$$- \frac{M}{\lambda J_t}\left(u(t-(\tau+\delta)) - \cos\lambda(t-(\tau+\delta))\right)$$

$t \geq \tau + \delta \Rightarrow$ After 1 Pulse

Define time such that $w_2(0) = 0$

$$w_1(t) = w_1(0)\cos\lambda t + \frac{M}{\lambda J_t}\left(\cos\lambda(t-(\tau+\delta)) - \cos\lambda(t-\tau)\right)$$

$$w_2(t) = -w_1(0)\sin\lambda t + \frac{M}{\lambda J_t}\left(\sin\lambda(t-\tau) - \sin\lambda(t-(\tau+\delta))\right)$$

$$\omega_t^2 = \omega_1^2 + \omega_2^2$$

$$\frac{\partial(\omega_t^2)}{\partial \tau} = 0 \Rightarrow \lambda\tau = \frac{\pi}{2} - \frac{\lambda\delta}{2}$$

Make pulse centered about negative peak of ω_2 to minimize wobble angle

$$\tan\theta = \frac{J_t \omega_t}{J_a n} \approx \theta$$

Choose δ such that $\lambda\delta = \pi$