

Torque-Free Axis-symmetric motion

$$0 = J_+ \dot{\omega}_1 - (J_+ - J_a) \omega_2 \omega_3$$

$$0 = J_+ \dot{\omega}_2 + (J_+ - J_a) \omega_1 \omega_3$$

$$0 = J_a \dot{\omega}_3$$

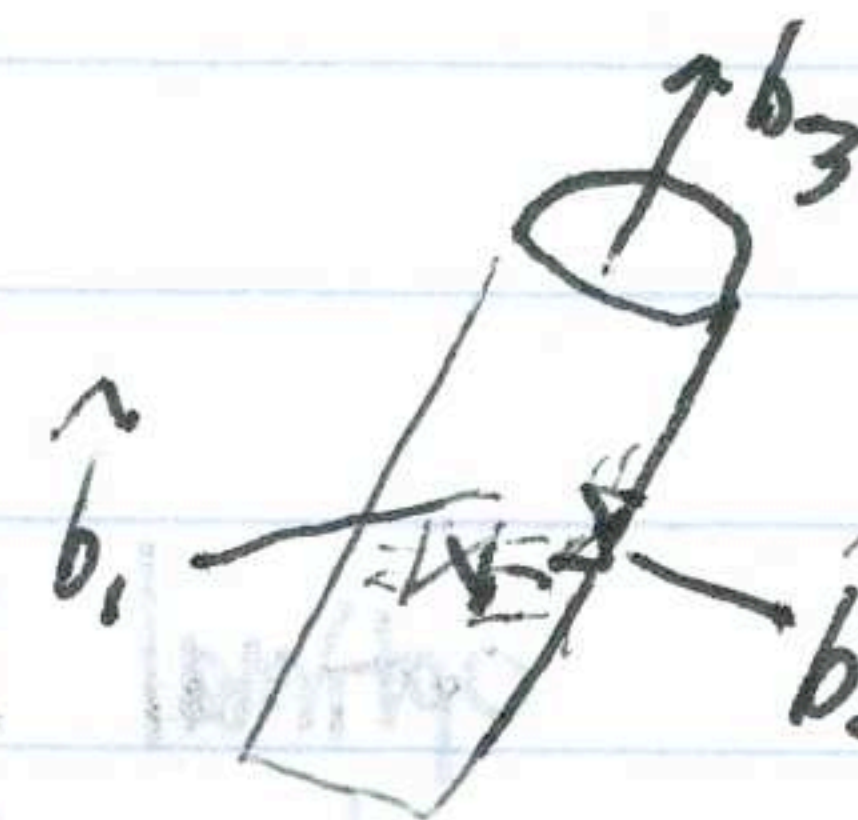
no longer Torque-Free:

$$M_1 = J_+ \dot{\omega}_1 - (J_+ - J_a) \omega_2 \omega_3$$

$$M_2 = J_+ \dot{\omega}_2 + (J_+ - J_a) \omega_1 \omega_3$$

$$\frac{M_1}{J_+} = \dot{\omega}_1 - \lambda \omega_2$$

$$\frac{M_2}{J_+} = \dot{\omega}_2 + \lambda \omega_1$$



$$M_1 = 0 \quad M_2 = M (u(t-\tau) - u(t-(\tau+\delta)))$$

$$M_2 = \begin{cases} 0 & t < \tau \\ M & \tau < t < \tau + \delta \\ 0 & t > \tau + \delta \end{cases}$$

$$\mathcal{L}(u(t)) = \frac{1}{s}$$

$$\mathcal{L}(u(t-\tau)) = \frac{1}{s} e^{-s\tau}$$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt$$

$$\mathcal{L}(f(t-\tau)) = \int_0^\infty e^{-st} f(t-\tau) dt$$

$$= \int_{-\tau}^\infty e^{-s(y+\tau)} f(y) dy$$

$$y = t - \tau \Rightarrow t = y + \tau \quad dy = dt \rightarrow$$

$$= e^{-s\tau} \underbrace{\int_0^\infty e^{-sy} f(y) dy}_{F(s)}$$

If both $M_1 + M_2 = 0$

$$\omega_1(t) = \omega_1(0) \cos(\lambda t) + \omega_2(0) \sin(\lambda t)$$

$$\omega_2(t) = \omega_2(0) \cos(\lambda t) - \omega_1(0) \sin(\lambda t)$$

$$\dot{y} + ky = u \quad y = \alpha_1 u_1 + \alpha_2 u_2 \rightarrow y = y_1 \alpha_1 + y_2 \alpha_2 + \text{IC response}$$

so we don't need to deal w/ IC any more

$$0 = s \omega_1 - \lambda \omega_2$$

$$\frac{M}{J_+} \frac{1}{s} = s \omega_2 + \lambda \omega_1$$

$$\omega_2 = \frac{1}{s} (-\lambda \omega_1 + \frac{M}{J_+} \frac{1}{s})$$

$$0 = s \omega_1 - \lambda \left(\frac{1}{s} (-\lambda \omega_1 + \frac{M}{J_+} \frac{1}{s}) \right) = s^2 \omega_1 + \lambda^2 \omega_1 - \frac{\lambda M}{J_+} \frac{1}{s}$$

$$\omega_1 = \frac{\lambda M}{J_+} \frac{1}{s(s^2 + \lambda^2)} = \frac{\lambda M}{J_+} \left(\frac{a}{s} + \frac{sb}{s^2 + \lambda^2} \right) \rightarrow \frac{\lambda M}{J_+} \frac{1}{s(s^2 + \lambda^2)} \quad a = -b$$

$$a = -b$$

$$\omega_1 = \frac{M}{\lambda J_+} \left(\frac{1}{s} - \frac{s}{s^2 + \lambda^2} \right) \quad \omega_1(t) = \frac{M}{\lambda J_+} (u(t) - \cos(\lambda t))$$

$$\omega_1(t) = \omega_1(0) \cos(\lambda t) + \omega_2(0) \sin(\lambda t) + \frac{M}{\lambda J_+} (u(t-\tau) - \cos(\lambda(t-\tau))) - \frac{M}{\lambda J_+} (u(t-(\tau+\delta)) - \cos(\lambda(t-(\tau+\delta))))$$

unit stops cancel $\omega_2(0) = 0 \rightarrow$

$$\omega_1(t) = \omega_1(0) \cos \lambda t + \frac{M}{\lambda J_+} (-\cos(\lambda(t-\tau)) + \cos(\lambda(t-(\tau+\delta))))$$

$$\omega_2(t) = -\omega_1(0) \sin \lambda t + \frac{M}{\lambda J_+} (\sin(\lambda(t-\tau)) - \sin(\lambda(t-(\tau+\delta))))$$

