

Asymmetric Spacecraft Spin Stability: J_1, J_2, J_3 and $\hat{\omega}^B = \omega_1 \hat{b}_1$

\Rightarrow perturbations $\hat{\omega}^B = (\omega_1 + \delta\omega_1) \hat{b}_1 + \delta\omega_2 \hat{b}_2 + \delta\omega_3 \hat{b}_3$ where $\omega_1 = \text{const.}$

$0 = J_1(\ddot{\omega}_1 + \delta\ddot{\omega}_1) - (J_2 - J_3)\delta\omega_2\delta\omega_3$ neglect products of small things

$\Rightarrow 0 = J_2\delta\ddot{\omega}_2 - (J_3 - J_1)\delta\omega_3(\omega_1 + \delta\omega_1) \Rightarrow 0 = J_2\delta\ddot{\omega}_2$

$0 = J_3\delta\ddot{\omega}_3 - (J_1 - J_2)(\omega_1 + \delta\omega_1)\delta\omega_2 \Rightarrow \delta\ddot{\omega}_1 = \text{const.}$

$0 = J_1\delta\ddot{\omega}_1$

$\Rightarrow 0 = J_2\delta\ddot{\omega}_2 - (J_3 - J_1)\delta\omega_3\omega_1 \Rightarrow 0 = \delta\ddot{\omega}_2 - \left(\frac{(J_3 - J_1)\omega_1}{J_2}\right)\delta\omega_3$

$0 = J_3\delta\ddot{\omega}_3 - (J_1 - J_2)\omega_1\delta\omega_2 \Rightarrow 0 = \delta\ddot{\omega}_3 - \left(\frac{(J_1 - J_2)\omega_1}{J_3}\right)\delta\omega_2$

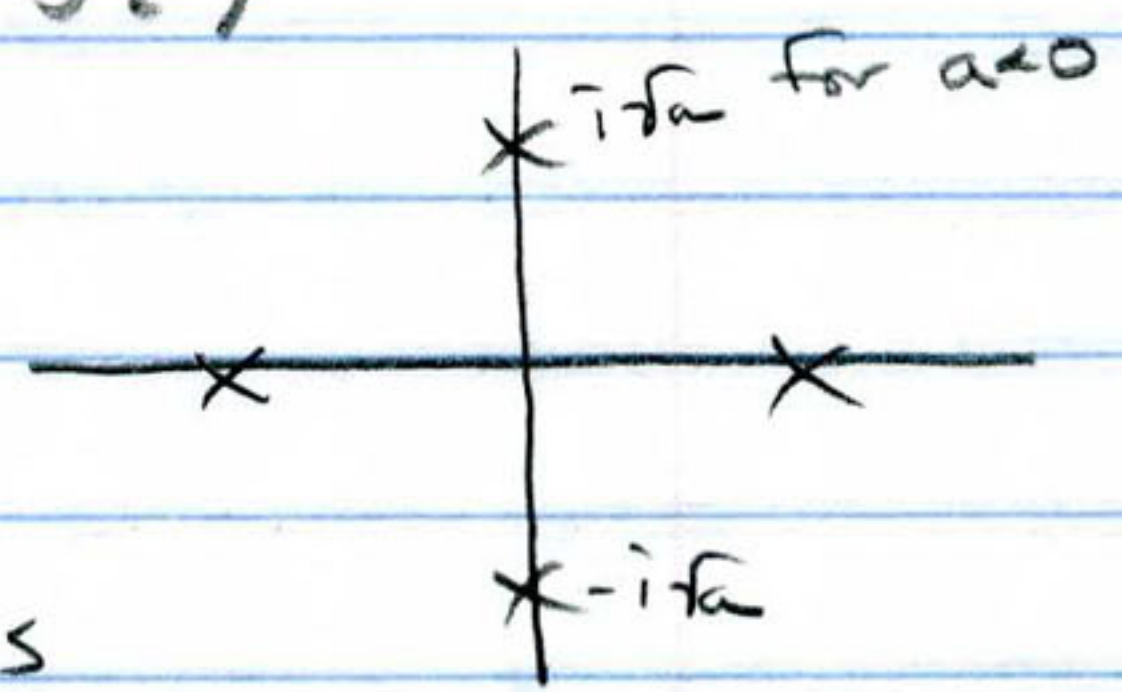
$\Rightarrow 0 = s\delta\omega_2 - \left(\frac{(J_3 - J_1)\omega_1}{J_2}\right)\delta\omega_3 - \delta\omega_2(0)$ Laplace Transform

$0 = s\delta\omega_3 - \left(\frac{(J_1 - J_2)\omega_1}{J_3}\right)\delta\omega_2 - \delta\omega_3(0)$

$\Rightarrow \delta\omega_3 = \frac{1}{s} \left[\left(\frac{(J_1 - J_2)\omega_1}{J_3}\right)\delta\omega_2 + \delta\omega_3(0) \right]$

$\Rightarrow 0 = s\delta\omega_2 - \frac{1}{s} \left(\frac{J_3 - J_1}{J_2}\right)\omega_1 \left(\frac{J_1 - J_2}{J_3}\right)\omega_1\delta\omega_2 - \frac{1}{s} \left(\frac{J_3 - J_1}{J_2}\right)\omega_1\delta\omega_3(0) - \delta\omega_2(0)$

$\Rightarrow \delta\omega_2(s) = \frac{\text{something}}{s^2 - \omega_1^2 \left(\frac{J_3 - J_1}{J_2}\right) \left(\frac{J_1 - J_2}{J_3}\right)} = \frac{\text{sum}}{s^2 - a}$



unstable when $J_3 > J_1$ and $J_1 > J_2$

\Rightarrow unstable when J_1 , intermediate axis, is spin axis

or $J_1 > J_3$ and $J_2 > J_1 \Rightarrow$ intermediate unstable \Rightarrow perturbations grow

If spinning about major/minor axis, perturbations don't grow