From Homework 5:

$$\frac{\partial \omega_1}{\partial \tau} = \omega_2 \qquad \frac{\partial \omega_2}{\partial \tau} = -\omega_1$$

$$\ddot{\omega}_1 = \dot{\omega}_2 = -\omega_1$$

Must de-couple before you can solve

Asymmetric spacecraft - Spin Stability

J_1, J_2, J_3

$$^A\vec{\omega}^B = \omega_1 \hat{b}_1$$

$$= (\omega_1 + \delta\omega_1)\hat{b}_1 + \delta\omega_2 \hat{b}_2 + \delta\omega_3 \hat{b}_3$$

Will perturbations grow or shrink?

Must satisfy Euler's Eqns (Torque-Free)

$$0 = J_1(\dot{\omega}_1^{\to 0} + \delta\dot{\omega}_1) - (J_2 - J_3)\,\delta\omega_2\,\delta\omega_3^{\to 0}$$

$$0 = J_2\,\delta\dot{\omega}_2 - (J_3 - J_1)\,\delta\omega_3(\omega_1 + \delta\omega_1)$$

$$0 = J_3\,\delta\dot{\omega}_3 - (J_1 - J_2)(\omega_1 + \delta\omega_1)(\delta\omega_2)$$

Assume perturbations are small

$\delta\omega_1 = \text{const}$

$$0 = J_2\,\delta\dot{\omega}_2 - (J_3 - J_1)\,\omega_1\,\delta\omega_3$$

$$0 = J_3\,\delta\dot{\omega}_3 - (J_1 - J_2)\,\omega_1\,\delta\omega_2$$

$$0 = \delta\dot{\omega}_2 - \left(\frac{(J_3 - J_1)\omega_1}{J_2}\right)\delta\omega_3$$

$$0 = \delta\dot{\omega}_3 - \left(\frac{(J_1 - J_2)\omega_1}{J_3}\right)\delta\omega_2$$

LaPlace it!

$$0 = s\,\delta\omega_2 - \left(\frac{(J_3 - J_1)\omega_1}{J_2}\right)\delta\omega_3 - \delta\omega_2(0)$$

$$0 = s\,\delta\omega_3 - \left(\frac{(J_1 - J_2)\omega_1}{J_3}\right)\delta\omega_2 - \delta\omega_3(0)$$

$$\delta\omega_3 = \frac{1}{s}\left[\frac{(J_1 - J_2)\omega_1}{J_3}\delta\omega_2 + \delta\omega_3(0)\right]$$

$$0 = s\,\delta\omega_2 - \frac{1}{s}\left(\frac{J_3 - J_1}{J_2}\right)\omega_1\left(\frac{J_1 - J_2}{J_3}\right)\omega_1\,\delta\omega_2$$

$$- \frac{1}{s}\left(\frac{J_3 - J_1}{J_2}\right)\omega_1\,\delta\omega_3(0) - \delta\omega_2(0)$$

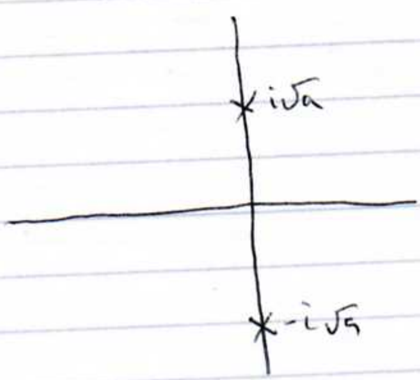
Only care about stability

$$\delta\omega_2(s) = \frac{\text{Some }\#}{s^2 - \underbrace{\omega_1^2\left(\frac{J_3 - J_1}{J_2}\right)\left(\frac{J_1 - J_2}{J_3}\right)}_{a}}$$

$a = -1$

$s^2 = -1$

$s = \pm i$



Unstable:

$J_3 > J_1$ and $J_1 > J_2$

$\hat{b}_1$ is intermediate axis

$\hat{b}_1$ must be either minor or major axis for stability

Exam

3 Questions

(dis) advantages between converting → eigenvalues => equiv. angles

- Representations [Euler angles, Rot matrices, quaternions]
- Angular Velocities
- Rigid Body Dynamics [Euler's eqns, axisymmetric, solve eqns of motion, torque-free w/initial conditions and w/constant torque(s), maybe time varying, active nutation control (complex)
  single step function, ICs, impulse